

**THE SOUTH FLORIDA REGIONAL
SIMULATION MODEL (SFRSM)**

**THE HYDROLOGIC SIMULATION ENGINE
(HSE)**

SOME REASONS BEHIND THE COMPLEXITY OF THE SYSTEM

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The South Florida hydraulic system is extremely complex and difficult to model because of its

- physical components,
- management operations,
- lack of data.

SOME REASONS BEHIND THE COMPLEXITY OF THE SYSTEM

- The system consist of a large number of complex components; some distributed, some lumped.
- The system has sparsely measured rainfall, ET and flow data at a number of locations.
- System is heavily managed. Even if management rules have been established, some parts of the system may still operate under unknown conditions.

SOME REASONS BEHIND THE COMPLEXITY OF THE SYSTEM

- Some of the most important parameters related to vegetation roughnes, and ET are not constant, and are related to the hydrology, ecology, weather, etc.
- The system has managed and unmanaged natural areas too close. Interactions of the components add to the complexity to the model. Assumptions used to derive governing PDE's are weak.
- The relative flatness of the region makes the system response very slow for natural areas and rapidly to urban areas simultaneously. Mathematically this leads to stiff differential equations.

IT APPEARS THAT...

- SFWMM/NSM are the only models that have captured the most important features of the system
- Any new model for South Florida should have a good backbone, and be ready to evolve with time.
- A model for South Florida will never be "done", but will grow and evolve with time.
- Many methods, technologies, and assumptions developed outside this area may not work, unless they are developed for the area.

SFRSM AND IT'S TOOLS

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The SFRSM model should be considered as one tool in a toolbox.

The tools include

- Pre-designed water bodies to carry out overland, groundwater and canal flows.
- Pre-designed water movers that can carry out o/l, g/w, canal, lake seepage, stream-aquifer interaction, etc.
- Pre-designed vertical solutions (source term solvers) to handle ET/ rainfall/ unsaturated flows, etc.

SFRSM AND IT'S TOOLS, CONT..

- Room for expansion of new water bodies, movers, and vertical solutions.
- Instructions and methods to calculate space and time descretizations.
- Methods to compute numerical errors (Lal, 2000, WRR).

SFRSM AND IT'S TOOLS, CONT..

- A battery of test problems to check the integrity of the model and the built in computer system libraries at all times.
- Instructions to use toe model.
- Tools for sensitivity analysis

SFRSM AND IT'S TOOLS, CONT..

- Tools to verify the controllability and the observability of a model application.
- Methods and tools for calibration (Lal, 1995, J. Hy, ASCE).

MODEL LIMITATIONS, ERRORS AND UNCERTAINTIES

MODEL LIMITATIONS, ERRORS AND UNCERTAINTIES

- Limitations due to weak assumptions:

The basic assumptions behind the development of the model not 100% accurate. (ET equations, Saint Venant equations, aquifer isotropy, ...)

- Limitations of the numerical algorithms:

(limitation of solution methods)

- Numerical errors due to discretization:

(use of rectangular grids on curved boundaries, reading too much into large cells)

MODEL LIMITATIONS, ERRORS AND UNCERTAINTIES, CONT..

- Limitations due to large model uncertainty

1. Input uncertainty
2. Parameter uncertainty
3. Algorithm uncertainty

SOME SOLUTIONS TO THESE PROBLEMS

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1. Scientifically verify the model results using observed data and analytical solutions.
2. Use (statistical) uncertainty bands to with model results.
3. Apply strict qualifications and conditions under which the results of the model could be used.
4. Use more than one model

CONVERGENCE OF THE MANY NEW TECHNOLOGIES HELPED HSE

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(A) Great algorithms

- Finite volume methods based on conservative forms of the governing equations (1972). (Simple to understand)
- Implicit solutions, that can be made unconditionally stable
- Error control mechanisms; can design codes to give error control (1998).

CONVERGENCE OF THE MANY NEW TECHNOLOGIES HELPED HSE, CONT..

(B) Object oriented methods, helped to design code that can grow systematically, keep data separately (1995 ANSI C++ standard)

- Data encapsulation
- Inheritance
- Metamorphism

CONVERGENCE OF THE MANY NEW TECHNOLOGIES HELPED HSE, CONT..

(C) Sparse solver technology (1997) helped to..

- Solve most implicit problems with grace, good diagnostics, etc.
- Speed of the code improve annually with ANL improving PETSC
- Speed computations by adaptively learning (algorithms). (eg. Program speeds up with no rains.)
- use MPI, parallel processing built in to PETSC
- use PETSC which also has other matrix, LP etc. algorithms.

CONVERGENCE OF THE MANY NEW TECHNOLOGIES HELPED HSE, CONT..

(D) Other IT related methods (STL, XML, Python, ..)

HSE in its current form was not possible before 1995

MODEL APPLICATIONS CAN BE DESIGNED

based on

ACCURACY AND RUN TIME

or

BENEFIT AND COST

References:

(Lal, Performance comparison of overland flow algorithms, ASCE HY 124(8), 1998)

(Lal, Numerical errors in overland flow and groundwater flow, WRR 36(5), 2000)

MODEL APPLICATIONS CAN BE DESIGNED

Numerical error (fully implicit/explicit):

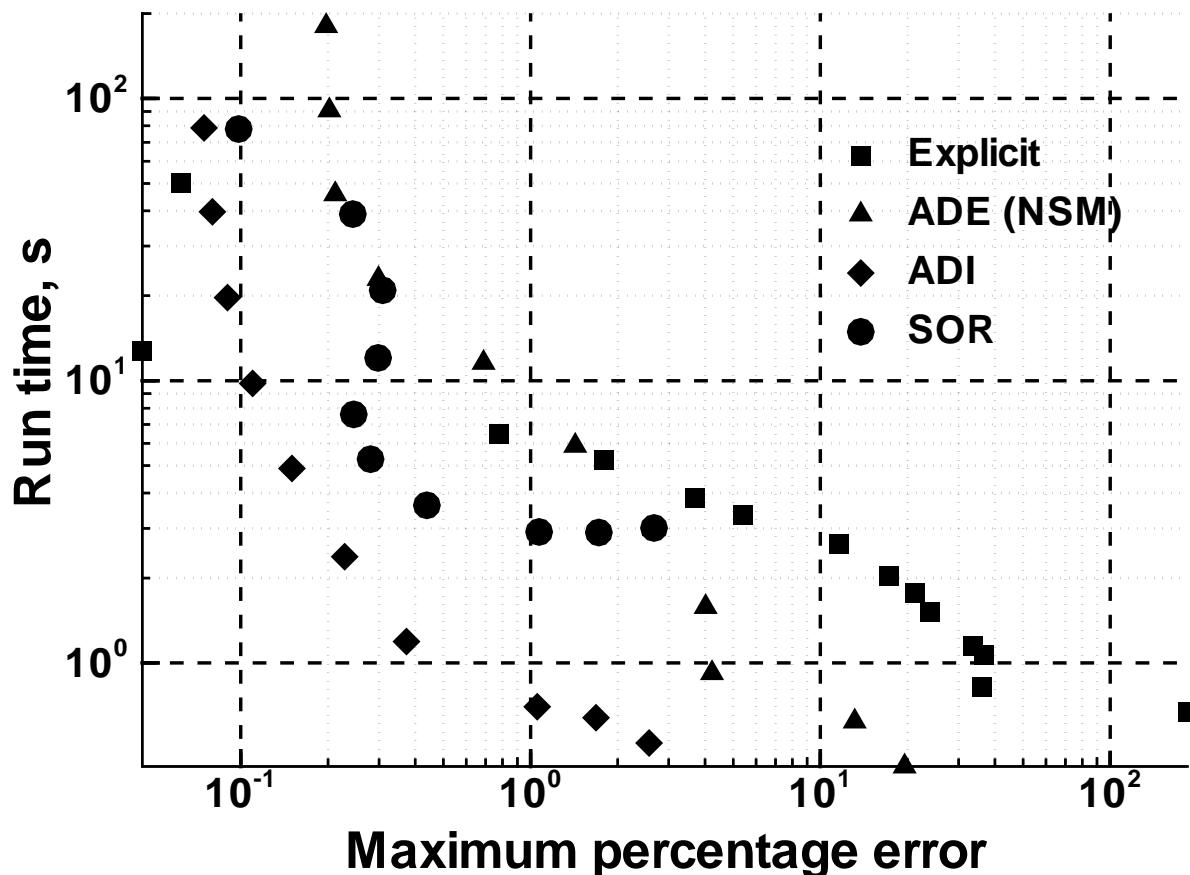
$$\varepsilon_T = 2TKk^2\phi^2\beta \quad (1)$$

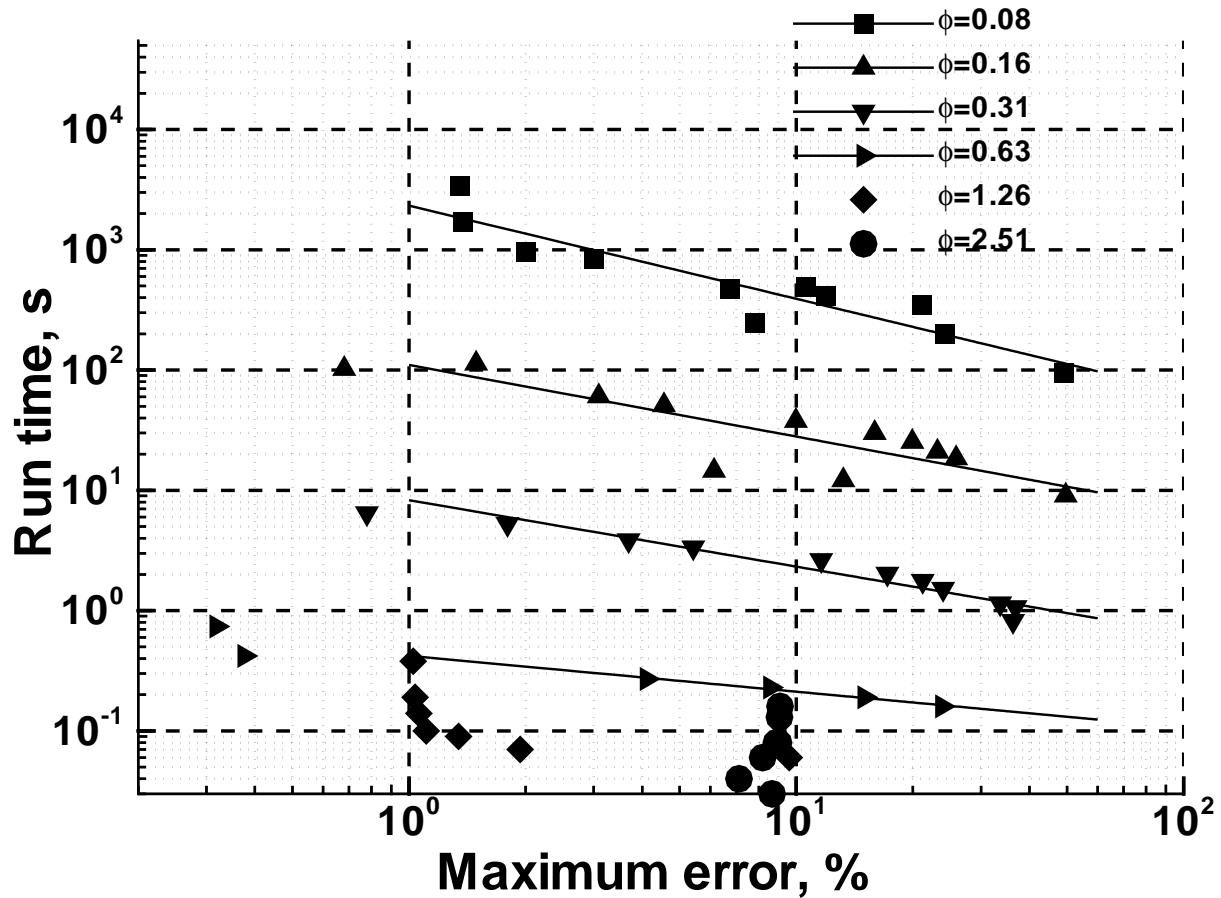
Run time:

$$t_r = c_1 \frac{TKAk^4}{\beta\phi^4} \quad (2)$$

ϕ = dimensionless spatial discretization

β dimensionless timestep





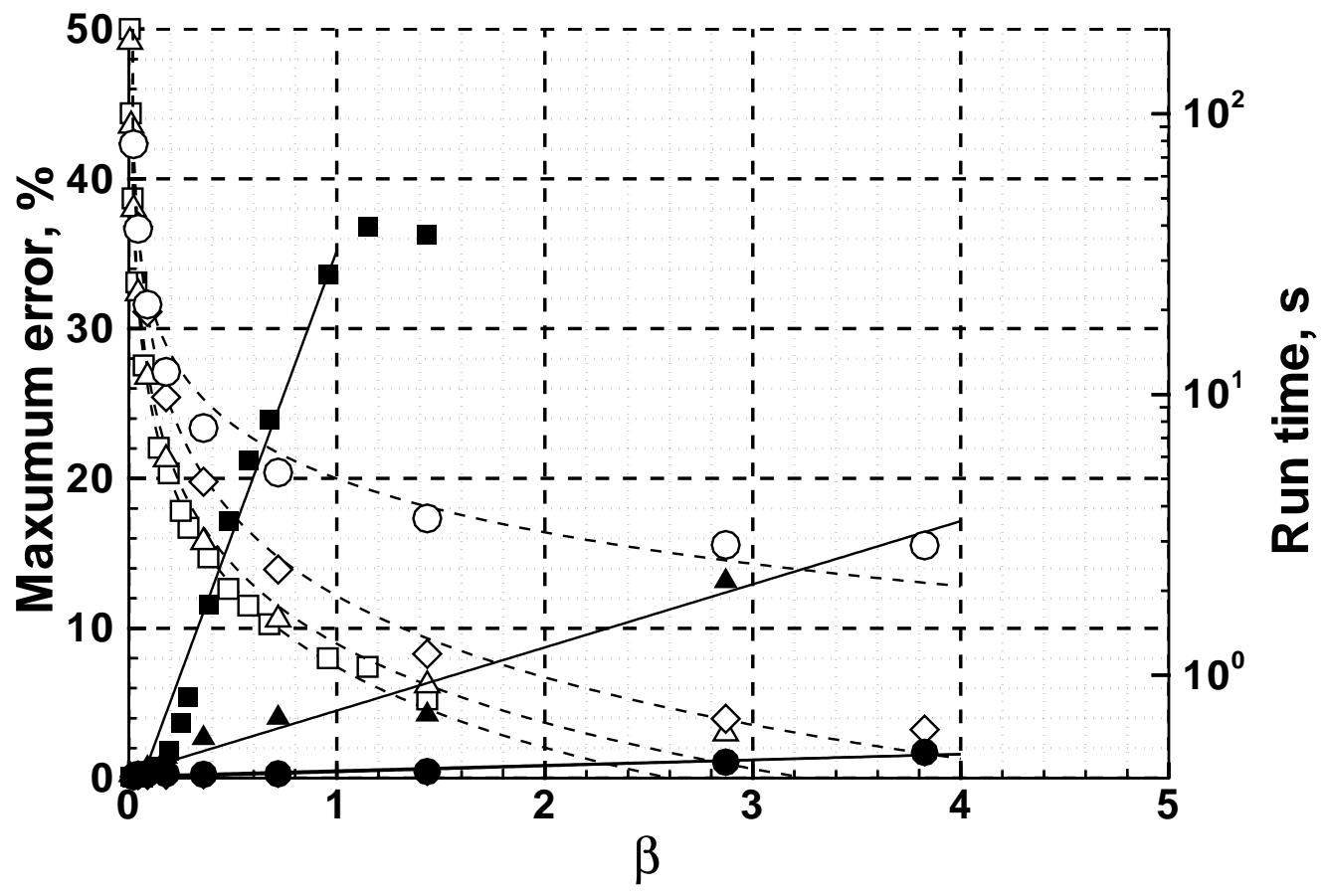


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GOVERNING EQUATIONS

CONSERVATIVE FORM

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} + \mathbf{S} = 0 \quad (3)$$

Water bodies

$$\mathbf{U} = \begin{pmatrix} h \\ uh \\ vh \\ c_i \end{pmatrix}, \quad (4)$$

Water movers

$$\mathbf{F} = \begin{pmatrix} uh \\ hu^2 + \frac{h^2 g}{2} \\ huv \\ uc_i \end{pmatrix}, \quad (5)$$

$$\mathbf{G} = \begin{pmatrix} vh \\ huv \\ hv^2 + \frac{h^2 g}{2} \\ vc_i \end{pmatrix}, \quad (6)$$

Vertical solutions

$$\mathbf{S} = \begin{pmatrix} q_l \\ gh(S_{fx} - S_{ox}) \\ gh(S_{fy} - S_{oy}) \\ f(c_i, \dots) \end{pmatrix} \quad (7)$$

WATER MOMOVERS, WATER BODIES AND CONSERVATIVE FORM

WATER MOMOVERS, WATER BODIES AND CONSERVATIVE FORM

U = water bodies

F, G = water movers

S = vertical solutions

ST VENANT EQUATIONS

Continuity equation:

$$\frac{\partial h}{\partial t} + \frac{\partial(hu)}{\partial x} + \frac{\partial(hv)}{\partial y} - RF + IN + ET + q_{ea} = 0 \quad (8)$$

Momentum equation:

$$\frac{\partial(hu)}{\partial t} + \frac{\partial(u^2h)}{\partial x} + \frac{\partial(uvh)}{\partial y} + hg \frac{\partial(h+z)}{\partial x} + ghS_{fx} = 0 \quad (9)$$

$$\frac{\partial(hv)}{\partial t} + \frac{\partial(uvh)}{\partial x} + \frac{\partial(v^2h)}{\partial y} + hg \frac{\partial(h+z)}{\partial y} + ghS_{fy} = 0 \quad (10)$$

INTEGRAL FORM OF THE MOMENTUM EQUATION:

$$\frac{\partial \mathbf{V}}{\partial t} + \nabla \left(\frac{1}{2} V^2 + gH \right) + g\vec{S}_f + \mathbf{V} \times \boldsymbol{\omega} = 0 \quad (11)$$

in which $\boldsymbol{\omega} = \nabla \times \mathbf{V}$;

$\mathbf{V} = u\mathbf{i} + v\mathbf{j}$ = velocity vector;

\vec{S}_f = friction slope vector;

$H = h + z$ = water level above the datum;

z = bottom elevation above datum.

DERIVATION OF DIFFUSION FLOW

$$u = -\frac{K}{h} \frac{\partial H}{\partial x}, \quad v = -\frac{K}{h} \frac{\partial H}{\partial y} \quad (12)$$

$$K = \frac{1}{n_b} h^{\gamma+1} S_n^{\lambda-1} \quad \text{for } \lambda \geq 1 \quad \text{and} \quad |S_n| > \delta_s \quad (13)$$

$$K = K_0 \quad \text{for } \lambda < 1 \quad \text{and} \quad |S_n| \leq \delta_s \quad (14)$$

The continuity equation can be expressed, using as

$$\frac{\partial H}{\partial t} = \frac{\partial}{\partial x} K \frac{\partial H}{\partial x} + \frac{\partial}{\partial y} K \frac{\partial H}{\partial y} + S \quad (15)$$

THE FINITE VOLUME METHOD

THE FINITE VOLUME METHOD

Continuity equation written as

$$\frac{\partial}{\partial t} \int_{cv} H \, dv + \int_{cv} \left[\frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) - S \right] \, dv = 0 \quad (16)$$

in which dv = volume of element cv .

The Gauss divergence theorem used to
simplify the volume integral term to a surface integral

$$\Delta \mathbf{A} \cdot \frac{d\mathbf{H}}{dt} = \int_{cs} \mathbf{H} \cdot \vec{n} \, dS + \mathbf{S} \quad (17)$$

THE MODEL TEMPLATE

$$\Delta \mathbf{A} \cdot \frac{d\mathbf{H}}{dt} = \mathbf{Q}(\mathbf{H}) + \mathbf{S} \quad (18)$$

in which water bodies, $\mathbf{H} = [H_1, H_2, \dots, H_m, \dots, H_{nc}]^T$

$$\mathbf{Q}(\mathbf{H}) = \mathbf{M} \cdot \mathbf{H} \quad (19)$$

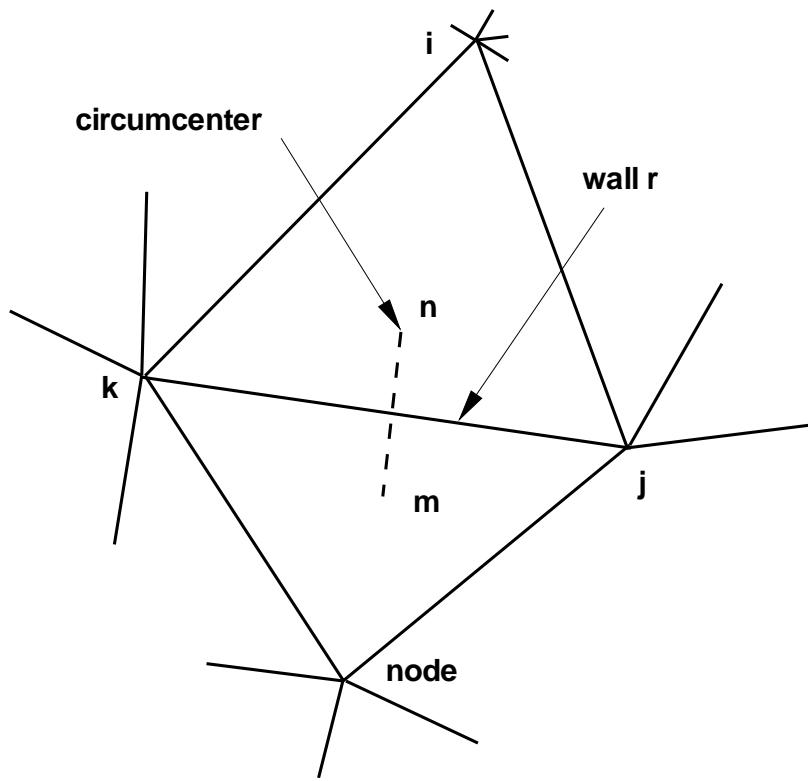
The net inflow rate to a cell m is given by

$$\Delta l_r \quad (20)$$

Δl_r = length of the side r

$\mathbf{n} = n_x \mathbf{i} + n_y \mathbf{j}$ = unit outward normal

THE CIRCUMCENTER-BASED METHOD FOR COMPUTING WALL
FLUX



THE CIRCUMCENTER-BASED METHOD FOR COMPUTING WALL FLUX

The current finite volume method used \approx A low-order mixed finite element method based on RT0 elements (Raviart and Thomas, 1977) (Cordes and Putti (1996)

THE CIRCUMCENTER-BASED METHOD FOR COMPUTING WALL FLUX

$(\hat{\mathbf{F}} \cdot \mathbf{n})_r$ for wall r is computed using

$$W_r = (\hat{\mathbf{F}} \cdot \mathbf{n})_r = K_{mn}(H_m - H_n) \quad (21)$$

in which, (13) or (85) is used to compute K_{mn} .

$$\begin{aligned} & \text{if } \{H_m > H_n \text{ and } h_m > 0 \text{ and } H_m > z_n\} \\ & \text{or } \{H_m < H_n \text{ and } h_n > 0 \text{ and } H_n > z_m\} \\ \text{then } K_{mn} &= \frac{h_r^{\frac{5}{3}}}{n_r \sqrt{S_r}} \text{ for } S_r > S_{tol} \end{aligned} \quad (22)$$

$$K_{mn} = \frac{h_r^{\frac{5}{3}}}{n_r \sqrt{S_{tol}}} \text{ for } S_r \leq S_{tol} \quad (23)$$

$$K_{mn} = 0 \text{ otherwise} \quad (24)$$

h_r and n_r are defined as

$$h_r = 0.5(h_m + h_n) \quad (25)$$

$$n_r = 0.5(n_m + n_n) \quad (26)$$

H_m, H_n = heads at the circumcenters; h_m, h_n = water depths at the circumcenters;

n_m, n_n = manning's roughness coefficients of cells m and n . S_r is computed using

$$S_r = \sqrt{\frac{(\hat{H}_j - \hat{H}_k)^2}{\Delta l_r^2} + \frac{(H_m - H_n)^2}{\Delta d_{mn}^2}} \quad (27)$$

FILL MATRIX

$$M_{m,n} \rightarrow M_{m,n} + \frac{K_{mn}\Delta l_r}{\Delta d_{mn}}, \quad M_{m,m} \rightarrow M_{m,m} - \frac{K_{mn}\Delta l_r}{\Delta d_{mn}} \quad (28)$$

FLOW VELOCITY

$$\vec{v} = \frac{1}{2A h} \left[Q_{s1} \begin{pmatrix} x - \hat{x}_1 \\ y - \hat{y}_1 \end{pmatrix} + Q_{s2} \begin{pmatrix} x - \hat{x}_2 \\ y - \hat{y}_2 \end{pmatrix} + Q_{s3} \begin{pmatrix} x - \hat{x}_3 \\ y - \hat{y}_3 \end{pmatrix} \right] = -K \nabla H \quad (29)$$

Here, Q_{s1}, Q_{s2}, Q_{s3} = discharge rates across cell walls $s1, s2$ and $s3$

FORMULATION OF THE WEIGHTED IMPLICIT METHOD

FORMULATION OF THE WEIGHTED IMPLICIT METHOD

$$\Delta A_i H_i^{n+1} = \Delta A_i H_i^n + \Delta t [\alpha Q_i^{n+1} + (1 - \alpha) Q_i^n] + \Delta t [\alpha S_i^{n+1} + (1 - \alpha) S_i^n] \quad (30)$$

system of linear equations:

$$[\Delta \mathbf{A} - \alpha \Delta t \mathbf{M}^{n+1}] \cdot \Delta \mathbf{H} = \Delta t [\mathbf{M}^n] \cdot \mathbf{H}^n + \Delta t (1 - \alpha) [\mathbf{M}^n - \mathbf{M}^{n+1}] \cdot \mathbf{H}^n \\ + \Delta t [\alpha \mathbf{S}^{n+1} + (1 - \alpha) \mathbf{S}^n] \quad (31)$$

GROUNDWATER

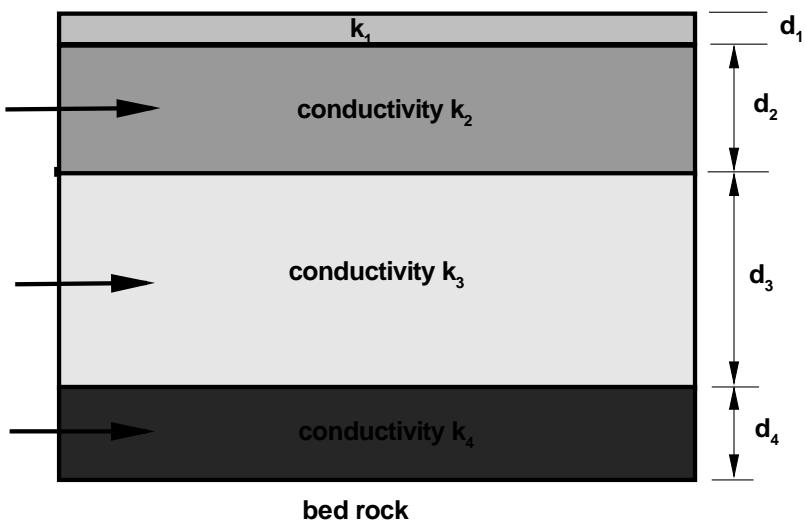


Figure 1: Layered aquifer structure

GROUNDWATER

Transmissivity:

$$T = \sum_{i=1}^{nl} k_i d_i \quad (32)$$

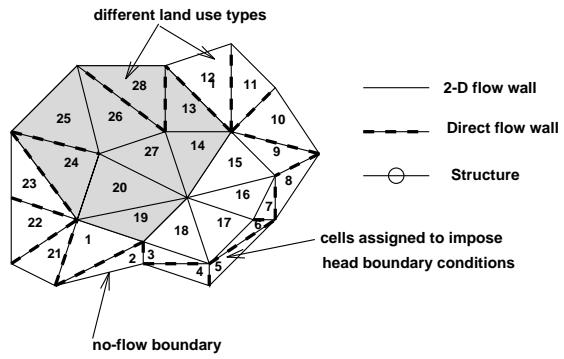


Figure 2: Definition sketch explaining cell and wall types

CANAL FLOW

GOVERNING EQUATIONS FOR CANAL FLOW

Continuity eq.:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} + q_{ea} = 0 \quad (33)$$

Momentum eq.:

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\beta \frac{Q^2}{A} \right) + gA \left(\frac{\partial H}{\partial x} + S_f \right) = 0 \quad (34)$$

DIFFUSION FLOW

$$Q = -K \frac{\partial H}{\partial x} \quad (35)$$

$$K = \frac{1}{n} AR^\gamma S_n^{\lambda-1} \quad \text{for } \lambda \geq 1, \quad |S_n| > S_{tol} \quad (36)$$

$$K = \frac{1}{n} \frac{AR^\gamma}{S_{tol}^{1-\lambda}} \quad \text{for } \lambda < 1, \quad |S_n| \leq S_{tol} \quad (37)$$

S_{tol} = a cutoff level of slope, $\approx 1.0 \times 10^{-10}$

FINITE VOLUME METHOD

$$\frac{\partial}{\partial t} \int_{cv} A dv - \int_{cv} \frac{\partial Q}{\partial x} dx + \int_{cv} q_{ae} dv = 0 \quad (38)$$

For a single canal segment i of plan area $B_i \Delta x_i$,

$$B \Delta x \frac{\partial H}{\partial t} = [\sum Q_{in} - \sum Q_{out} + q_{ae}] = 0 \quad (39)$$

NUMERICAL IMPLEMENTATION

$$Q = K_{i,j}(H_i - H_j) \quad (40)$$

in which

$$\begin{aligned} & \text{if } H_i > H_j \text{ and } H_i > z_j \text{ and } H_i > z_i \\ & \text{or } H_j > H_i \text{ and } H_j > z_i \text{ and } H_j > z_j \\ \text{then } K_{ij} &= \frac{\bar{A}}{(l_i + l_j)\sqrt{S_n}\bar{n}_b} \left(\frac{\bar{A}}{\bar{P}} \right)^{\frac{5}{3}} \text{ if } S_n > S_{tol} \end{aligned} \quad (41)$$

$$\text{then } K_{ij} = \frac{\bar{A}}{(l_i + l_j)\sqrt{S_{tol}}\bar{n}_b} \left(\frac{\bar{A}}{\bar{P}} \right)^{\frac{5}{3}} \text{ if } S_n \leq S_{tol} \quad (42)$$

$$\text{otherwise } K_{ij} = 0 \quad (43)$$

in which,

$$\bar{A} = A_i + A_j \quad (44)$$

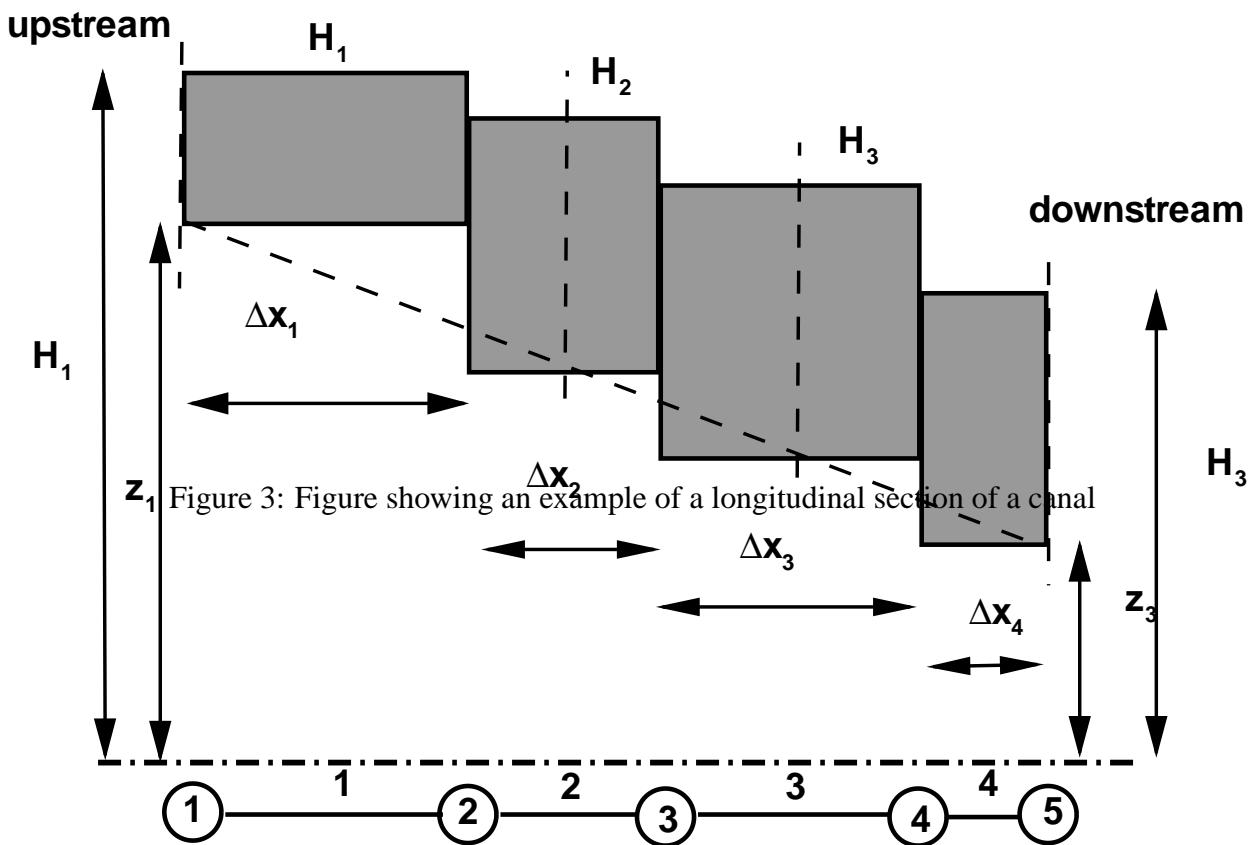
$$\bar{P} = P_i + P_j \quad (45)$$

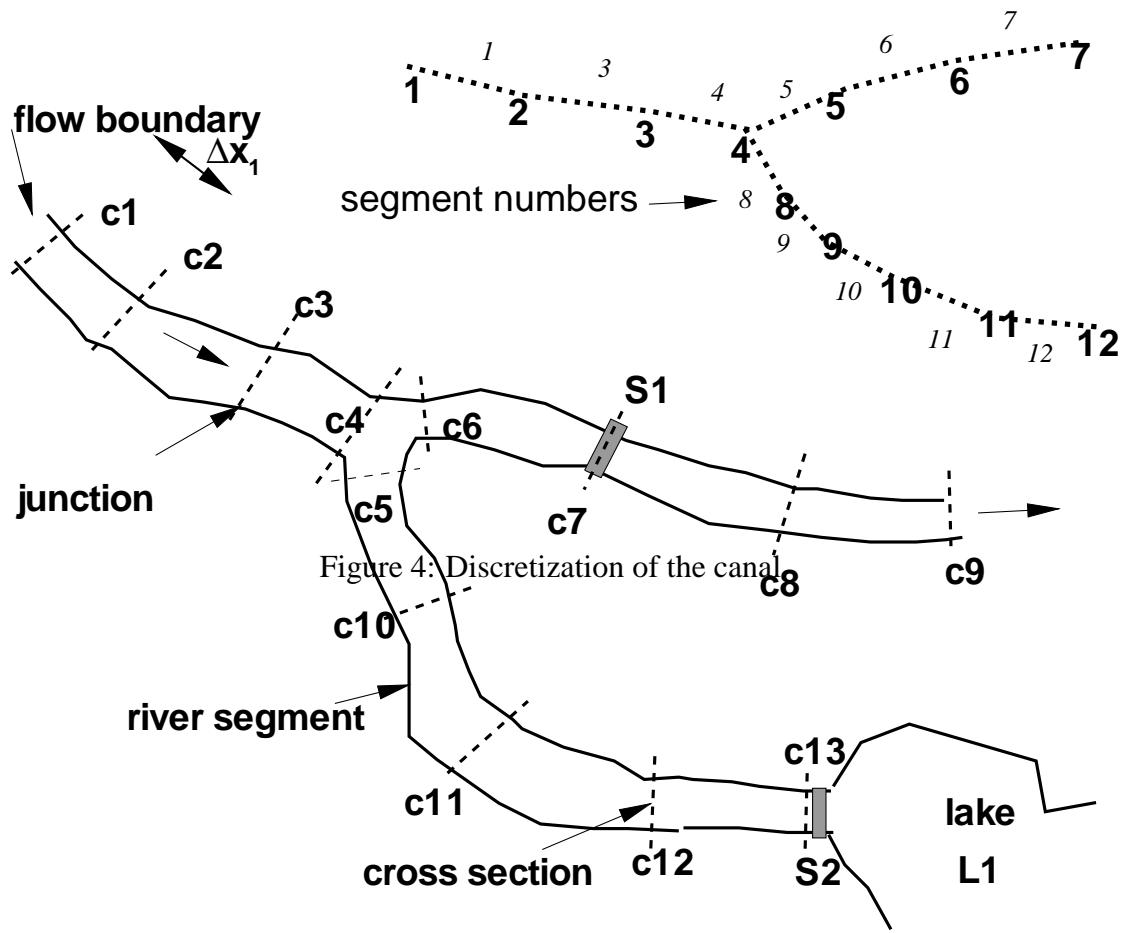
$$\bar{n}_b = n_{bi} + n_{bj} \quad (46)$$

$$S_n = \frac{|H_i - H_j|}{l_i + l_j} \quad (47)$$

$$l_i = \Delta x_i \text{ for segments with head b.c.} \quad (48)$$

$$l_i = 0.5\Delta x_i \text{ for all other segments.} \quad (49)$$





NUMERICAL IMPLEMENTATION OF THE CONTINUITY EQUATION

$$B_i \Delta x_i \frac{\partial H_i}{\partial t} = \left[\sum_{j=1}^{nd} K_{ij} H_j + q_{ae,i} \right] = 0 \quad (50)$$

Template equation:

$$\mathbf{M}\mathbf{A} \cdot \frac{d\mathbf{H}}{dt} = \mathbf{M} \cdot \mathbf{H} + \mathbf{S} \quad (51)$$

Fill matrix:

$$M_{ij} \rightarrow M_{ij} + K_{ij} \quad (52)$$

$$M_{ji} \rightarrow M_{ji} + K_{ij} \quad (53)$$

$$M_{ii} \rightarrow M_{ii} - K_{ij} \quad (54)$$

$$M_{jj} \rightarrow M_{jj} - K_{ij} \quad (55)$$

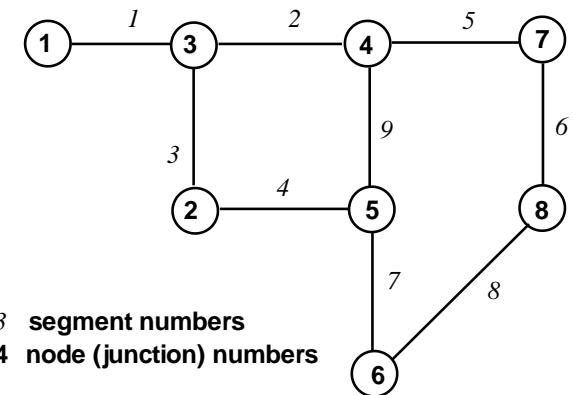


Figure 5: Figure showing an example of a discretization

Table 1: Sample data set for the network in Fig 5

CONNECTIVITY INFORMATION

Node Segments attached.

1	1
2	3 4
3	1 2 3
4	2 5 9
5	4 7 9
6	7 8

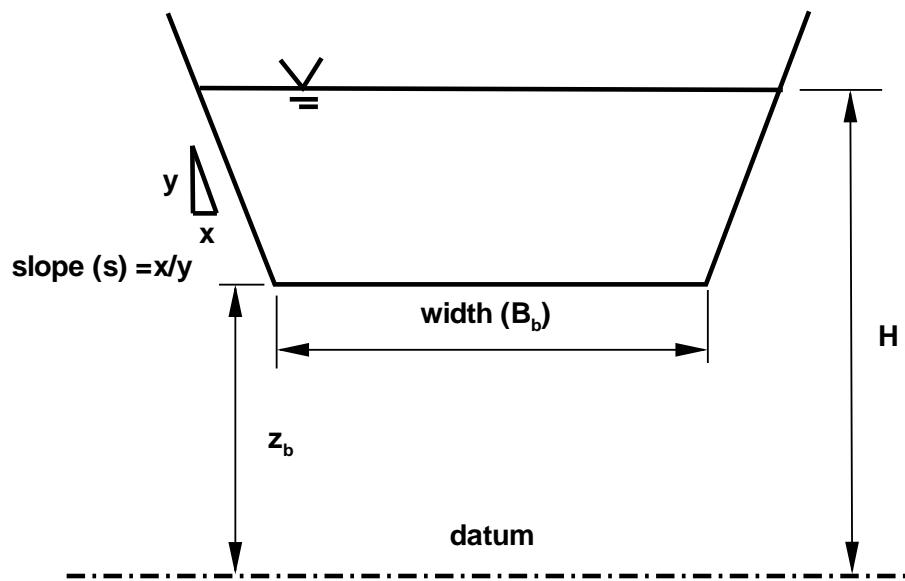
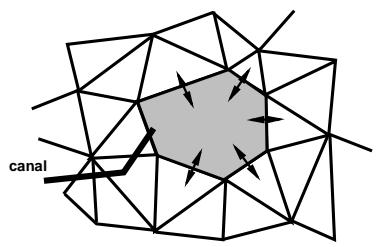


Figure 6: Cross section of a trapezoidal canal

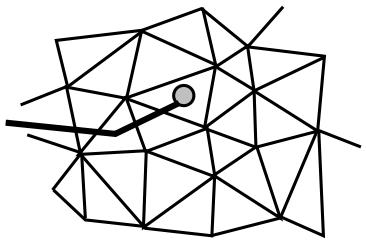
DATA FOR THE CANAL SEGMENT

```
ARC  
type trapezoid 100.0 498.0 0.0 0.2  
gwcoeff 1.0e-7  
ID 1  
NODES      1          2  
END
```

LAKES AND PONDS



(a) Large lake



(b) Surficial pond

SEEPAGE INTO LAKES AND PONDS

$$Q_{L,k} = l_w K_{L,k} (H_L - H_k) \quad (56)$$

equation of mass balance

$$\Delta A_L \frac{dH_L}{dt} = \sum_{k=1}^{nd} K_{L,k} l_w (H_k - H_L) \quad (57)$$

LAKEDATA

IL LAREA KVEG HINIT HBOT

LTYPE ICELL

NLOOKUP

STAGE_1 VOLUME_1

STAGE_2 VOLUME_2

....

STAGE_NLOOKUP VOLUME_NLOOKUP

LAKE EXAMPLE

1 6.0E6 1.0 504.0 400.0

lake 1

3

0.0 0.0

400.0 3.0e8

600.0 10.0e8

2 2.0E6 1.0 505.0 400.0

pond 3

4

0.0 0.0

400.0 4.0e8

500.0 10.0e8

600.0 12.0e8

URBAN ROUTING

using

USING THE MULTI BASIN ROUTING MODEL (MBR, CASCADE)

MBR FOR URBAN ROUTING

Characterization of urban catchments: eg.

URBAN CELL NO: 5

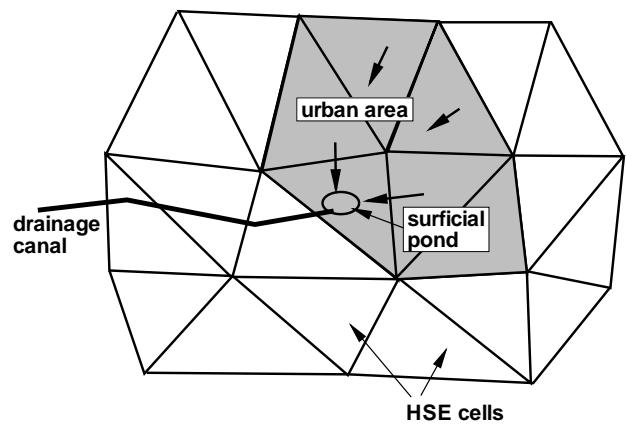
AREA: 200

GW STORAGE: 0.01

TIME OF CONCENTRATION: 5 HRS

CONSTITUENT HSE CELLS: 13 54 26 93 43

RELEASE POINT IN HSE: CELL 23



BASIC SCS ALGORITHM

```
Srf = Srf + Rf;  
if (Srf > Ia) Inf = Ia;  
else Inf = Srf;  
Srf = Sf - Inf;
```

Rf= rainfall for the duration.

Sf= cumulative rainfall,

Ia= maximum value of initial abstraction, assumed as 0.2 Si

S= potential maximum basin storage,

Inf= actual initial abstraction. The potential runoff depthi

BASIC SCS ALGORITHM, CONT..

$$r_o = Srf * Srf / (Srf + S)$$

r_o is computed using the cumulative value of Srf .

F = total infiltration or groundwater recharge

$$F = S * Srf / (Srf + S)$$

RUNOFF

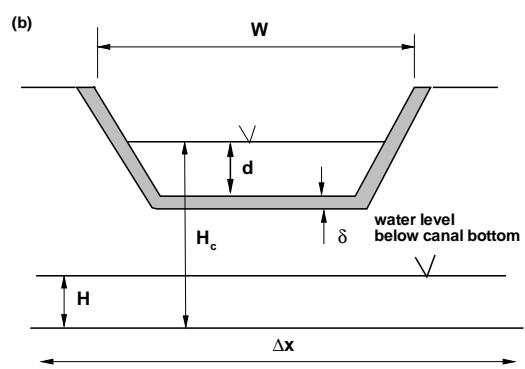
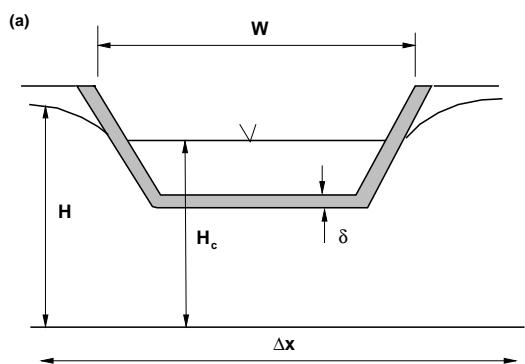
```
Q = RHO * Q_prev + DELRHO * (ro-ro_prev)
```

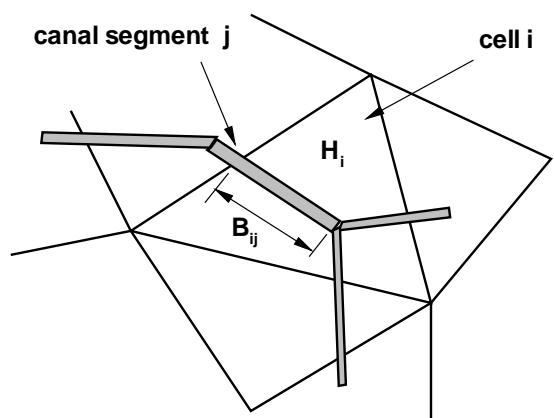
where

```
RHO = exp(-delt/tc)
```

```
DELRHO = RHO*(1.0/RHO -1.0)/delt
```

INTERACTION OF GROUNDWATER FLOW AND CANAL FLOW





EQUATION OF MASS BALANCE FOR LEAKAGE INTO A CELL

$$\Delta A_i \frac{dH_i}{dt} = \sum_{j=1}^{n_i} B_{ij} G_{ij} (H_{cj} - H_i) \quad (58)$$

ΔA_i = area of cell i .

This equation adds the following terms to the matrix.

$$M_{i,j} \rightarrow M_{i,j} - B_{ij} G_{ij} \quad (59)$$

$$M_{i,j} \rightarrow M_{i,j} + B_{ij} G_{ij} \quad (60)$$

EQUATION GOVERNING OVERLAND FLOW INTO A CANAL

for $H > H_c, \quad H - z - d_d > 0, \quad H > z_c, \quad H_c \leq z + d_d$

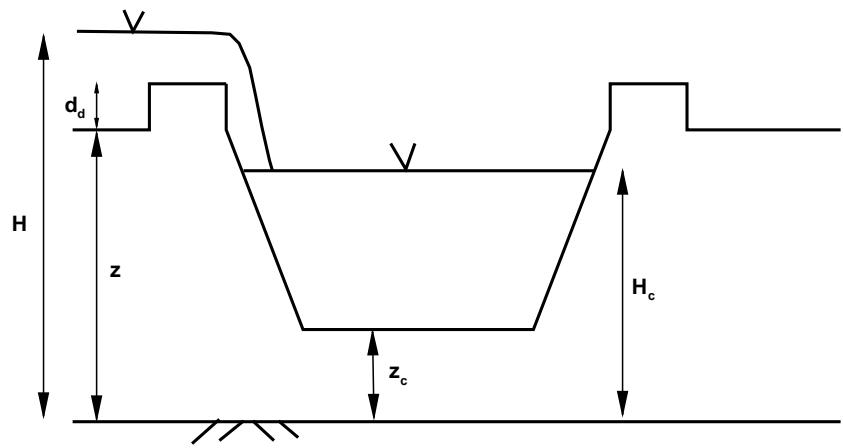
$$Q_o = \frac{1}{n_b} \sqrt{S_0} (H - z - d_d)^{\frac{5}{3}} \quad (61)$$

(62)

for $H > H_c, \quad H - z - d_d > 0, \quad H > z_c, \quad H_c > z + d_d$

$$Q_o = \frac{1}{n_b} (H - z - d_d)^{\frac{5}{3}} \sqrt{\frac{4(H - H_c)}{\Delta x}} \quad (63)$$

$Q_o = 0$ under all other conditions.



100

MODIFICATION OF THE MATRIX

$$M_{nc+j,i} \rightarrow M_{nc+j,i} + K_{int} \quad (64)$$

$$S_{nc+j} \rightarrow S_{nc+j} - K_{int}(z_i + d_d) \quad (65)$$

$$M_{i,i} \rightarrow M_{i,i} - \frac{K_{int}}{s_c} \quad (66)$$

$$S_i \rightarrow S_i + \frac{K_{int}}{s_c}(z_i + d_d) \quad (67)$$

THE STEADY STATE SOLUTION

obtained by using $\frac{\partial \mathbf{H}}{\partial t} = 0$

iterative solution method.

$$[-\mathbf{M}^n] \cdot \Delta \mathbf{H} = \mathbf{M}^n \cdot \mathbf{H}^n + \mathbf{S}^n \quad (68)$$

\mathbf{H}^n is updated using $\mathbf{H}^{n+1} = \mathbf{H}^n + \mathbf{H}^n$

THE CONVERGENCE CRITERIONS HAS TO BE SPECIFIED AS

$$||\mathbf{H}^n||_\infty < \delta_H \quad (69)$$

and

$$||\mathbf{M}^n \cdot \mathbf{H}^n + \mathbf{S}^n||_\infty < \delta_Q \quad (70)$$

NUMERICAL ERRORS

Ref: Lal, WRR 36(5), May 2000

In groundwater and diffusion flow, frequency f and wave number k of a solution are related by

$$f = \frac{K_g k^2}{s_c} \quad (71)$$

Δx is used to represent $k = 2\pi/L$ and Δt is used to represent $f = 2\pi/T$

REPRESENTATIONAL ERRORS

introduced when

- (a) Δx is inadequate to represent the solution accurately
- (b) Δt is inadequate to represent the solution accurately.

COMPUTATIONAL ERRORS

introduced when errors (a) and (b) in space and time undergo the computational process.

Table 2: Conditions needed to control errors

Condition	Purpose
$\phi < 0.5\sqrt{\epsilon_d}$	(a) To limit 1-D spatial representational errors to $\epsilon_d\%$
$\psi < 0.5\sqrt{\epsilon_d}$	(b) To limit 1-D temporal representational errors to $\epsilon_d\%$
$\phi^2\beta < 0.14$	(c) To limit computational errors to 5%; (Equivalent to $\psi < 0.14$ for small values)

$$\phi = k\Delta x,$$

$$\psi = f \Delta t;$$

$$\beta = K_g \Delta t / (s_c \Delta x^2);$$

Table 3: Practically useful formulas for numerical models

Equation	Reference
$X \sqrt{(fs_c/K)} = 4.3$	X is the distance at which a 1-D disturbance of frequency f would decay to 5% of the amplitude.
$\Delta x = 1.1 \sqrt{K/f}$	Δx gives the spatial discretization needed to represent a water surface profile with 5% accuracy. The profile is created by a disturbance of frequency f .
$\Delta x = 0.5 \sqrt{K\epsilon_d/f}$	Δx needed to represent the same spatial discretization with a ϵ_d % accuracy.

Equation	Reference
$K\Delta t/\Delta x^2 = 0.14$	Δt gives the time step needed if the numerical error is limited to 5% of the disturbing amplitude.
$\Delta x \sqrt{(fs_c/K)} < 1.4$	Δx gives the size of a square cell needed to solve the amplitude of a well fluctuation with a maximum error of 5%.
$\Delta x \sqrt{(f/K)} = 5$	gives a practically useful upper bound of Δx that can be used to model a pumping well (error < 40%).
$r \sqrt{(fs_c/K)} = 2.75$	r is the radius at which the amplitude of a well with $\hat{r}_w = 0.5$ decays to 5% of the amplitude of the well.
$\varepsilon = 2.07 \exp(-0.726r/\Delta x)$	ε gives the numerical error of a steady state ¹⁰⁹ well as a percentage of the drawdown.

Table 4: Variation of ϵ_w , the error in the amplitude of the cell containing a well

$\Delta x \sqrt{(fs_c/K)}$	0.01	0.02	0.05	0.1	0.2	0.5
C_c	1.0000	0.9999	0.9997	0.9990	0.9967	0.9844
ϵ_w (%)	$4.200 \cdot 10^{-7}$	$5.130 \cdot 10^{-5}$	$1.351 \cdot 10^{-4}$	$1.508 \cdot 10^{-3}$	$1.564 \cdot 10^{-2}$	0.289

$\Delta x \sqrt{(fs_c/K)}$	1	2	5	10	20
C_c	0.9523	0.8886	0.5640	0.2563	$4.3228 \cdot 10^{-2}$
ϵ_w (%)	2.13	11.1	43.6	74.4	95.7

SPATIAL AND TEMPORAL EXTENTS OF TRANSIENT MODELS

$$\frac{s_c X^2}{K T_p} = \frac{(\ln \alpha_d)^2}{\pi} \quad (72)$$

Table 5: Table of parameter values used for the L31N and Snapper Creek extension (SNE) canals in South Florida

Variable	$T_p = 1$ day	$T_p = 7$ days	$T_p = 30$ days
Λ (m)	57280	151550	313750
P_r	2.703×10^{-5}	2.703×10^{-5}	2.703×10^{-5}
P_b	2.339×10^{-3}	8.842×10^{-4}	4.274×10^{-4}
P_m	2.181×10^{-2}	5.771×10^{-2}	1.194×10^{-1}
P_d	26.8	10.1	4.9
$\exp(-\sqrt{0.5P_r}/P_m)$	0.845	0.938	0.969
$\hat{\lambda}_1$	1.915	3.113	4.452
$\hat{\lambda}_2$	0.821	1.311	1.884
$\hat{\mu}_1$	136	136	136
$\hat{\mu}_2$	136	136	136
ΔL (m)	7674	12716	18318
Δy (m)	46	123	254

CALIBRATION METHODS

Generalized linear inverse method (SVD)

Minimax method

Conjugate gradient method

(*Lal, J Hyd ASCE 212(9), Calibration of riverbed roughness*)

BUILDING BLOCKS OF THE MODEL

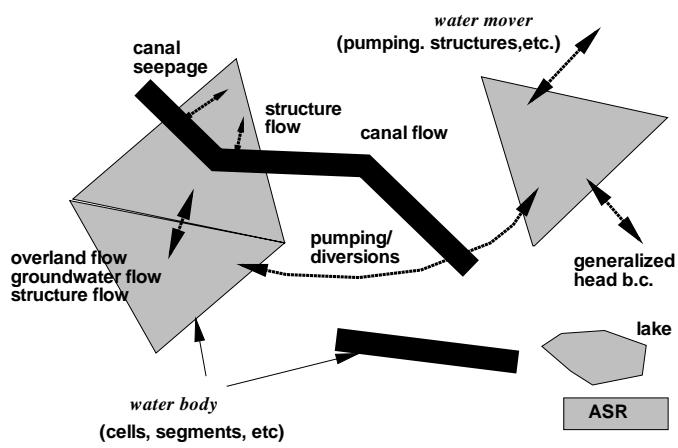


Figure 7: Water bodies and water movers in the model

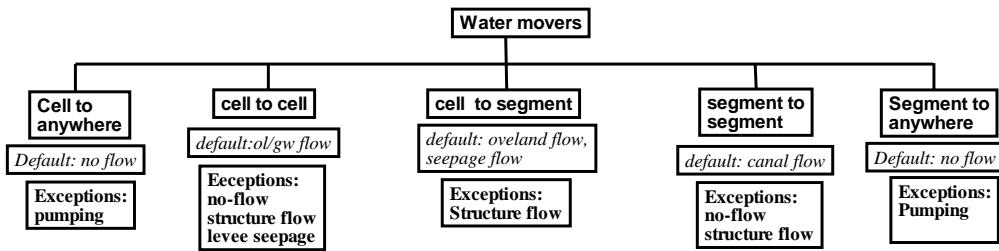


Figure 8: Possible flow paths in the model

Table 6: Types of basic flow types possible between two different water bodies

Between	control var.	Example
cell-cell	head diff.	if adjacent, the default is overland flow and groundwater flow.
segment-segment	head diff.	if adjacent, the default is canal flow.
cell-seg	head diff.	if adjacent, the default is canal leakage.
lake-cell	head diff.	if adjacent, lake seepage
any-any	any	water movers externally defined.

SAMPLE BOUNDARY CONDITION DATA

```
NumSourceBounds 2
26 dss m_canal.dss "/L8/L8.4/FLOW/01JAN1994/1DAY/M CAN/" * -0.0283
1 dss "l8_flw_stg.dss" "/L8/M CNL/FLOW//1DAY/15-WEST PALM CITY'S M/"

NumHeadBounds 2
1 dss l8_flw_stg.dss "/L8/L8.441/STG//1DAY/28 HWY 441 N/" * .3048
30 dss S5AS_data.dss "/C51W/S5AS_H/STG//1DAY/6RVATION AR/" * .3048
#30 dss S5AS_data.dss "/C51W/S5AE_H/STG//1DAY/48 CULVERT/" * .3048

NumNetWallBounds 1
41 40 uniformflow -0.0015
```

ADDITIONAL WATER MOVERS-PIPES

```
NumPipes    2
5 21 3.762  212.3  0.012  0.914  0.914  norev
14 20 3.762  212.3  0.012  0.914  0.914  rev
```

120

MBR WEIRS

NumMBRWiers 5

```
1 10 WeirBroad 500.5 100.0 0.01
2 11 WeirSharp 500.5 100.0
2 11 WeirSharp 500.0 200.0
2 12 WeirSharp 499.3 78.0
12 3 WeirDrop 500.5 2.0
```

MBR BLEEDERS

NumBldrs 3

1 10 BldrVNotch 500.5 501.0 30.0
2 12 BldrCircular 500.1 2.0
2 11 BldrRecta 501.0 0.4 0.6

Examples for output options

Example for cell head:

```
element 599 head dss L8out.dss "/hse/Dupuis1/head//1day/calc/"
```

Example for canal segment head:

```
Prism 1 Head dss can_out.dss "/hse/pind c01/head//5min/calc/"
```

Example for canal flowpair discharge:

```
flowpair 25 26 flow dss can_out.dss "/hse/pind f25/flow//5min/calc/"
```

Example for cell wall discharge:

```
wall 263 264 olflow dss L8out.dss "/hse/E425 <-> E534/olflow//1day/calc/"
```

ANALYTICAL SOLUTION OF THE STREAM-AQUIFER INTERACTION

GOVERNING EQUATIONS

$$s_c \frac{\partial H}{\partial t} = \frac{\partial}{\partial x} \left(K_g \frac{\partial H}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_g \frac{\partial H}{\partial y} \right) \quad (73)$$

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} - q_l = 0 \quad (74)$$

$$\frac{\partial h}{\partial x} + S_f - S_0 = 0 \quad (75)$$

LEAKAGE

$$S_f = C \frac{u^n}{h^m} = C \frac{q^n}{h^{m+n}} \quad (76)$$

$$q_l = \frac{2K_g}{B} \left(\frac{\partial H}{\partial y} \right)_{y=\delta+} \quad (77)$$

METHOD OF SOLUTION

Perturbation form

$$\frac{\partial}{\partial x} \left(K_g \frac{\partial H^*}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_g \frac{\partial H^*}{\partial y} \right) = s_c \frac{\partial H^*}{\partial t} \quad (78)$$

$$\frac{\partial h^*}{\partial t} + \frac{\partial q^*}{\partial x} - \frac{2K_g}{B} \left(\frac{\partial H^*}{\partial y} \right)_{y=\delta_+} = 0 \quad (79)$$

$$\frac{\partial h^*}{\partial x} + S_f \left\{ n \frac{q^*}{q_0} - (m+n) \frac{h^*}{h_0} \right\} = 0 \quad (80)$$

SOLUTION FORM

canal: $h^*(x, t) = h' \exp\{(ft + \lambda x)\}$ (81)

sediment: $H^*(x, y, t) = h' \exp(ft + \lambda x + \theta y)$ for $0 < y \leq \delta$ (82)

aquifer: $H^*(x, y, t) = h' \exp(ft + \lambda x + \theta \delta + \mu(y - \delta))$ for $y > \delta$ (83)

COMPLEX VARIABLE FORM

$$K_g(\lambda^2 + \mu^2) = s_c f \quad (84)$$

$$fh' - \frac{2K_g}{B}\mu e^{\delta\theta}h' + \lambda q' = 0 \quad (85)$$

$$\lambda h' - (m+n)\frac{S_f}{h_0}h' + \frac{S_fnq'}{q_0} = 0 \quad (86)$$

DIMENSIONLESS VARIABLES

$$\Lambda = \sqrt{\frac{q_0}{nS_f f_r}} \approx \sqrt{\frac{K_c}{n f_r}} \quad (87)$$

$$K_c = \frac{q}{S_f} \quad (88)$$

$$P_r = \frac{K_g}{s_c f_r \Lambda^2} = \frac{n K_g S_f}{s_c q_0} = \frac{n K_g}{s_c K_c} \quad (89)$$

$$P_b = \frac{B}{\Lambda s_c} \quad (90)$$

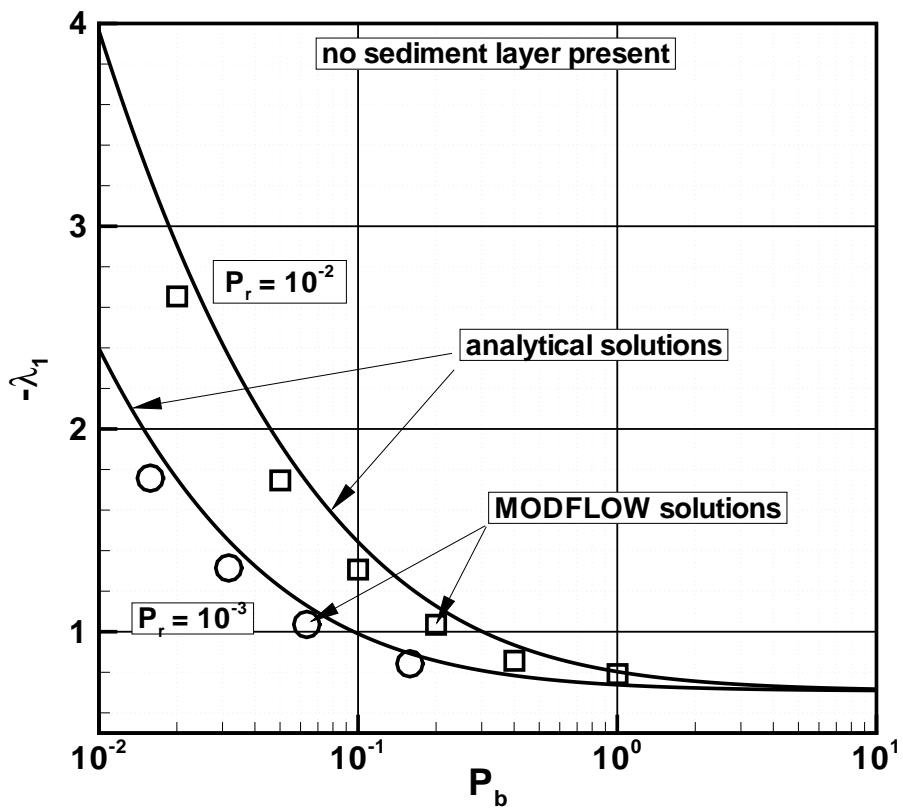
$$P_d = \frac{h_0}{(m+n)S_f \Lambda} \quad (91)$$

$$P_m = \frac{K_m}{\delta f_r \Lambda s_c} = \left(\frac{k_m}{\delta}\right)\left(\frac{1}{f_r}\right)\left(\frac{B}{\Lambda}\right)\left(\frac{1}{s_c}\right) \quad (92)$$

Dimensionless gov. eq:

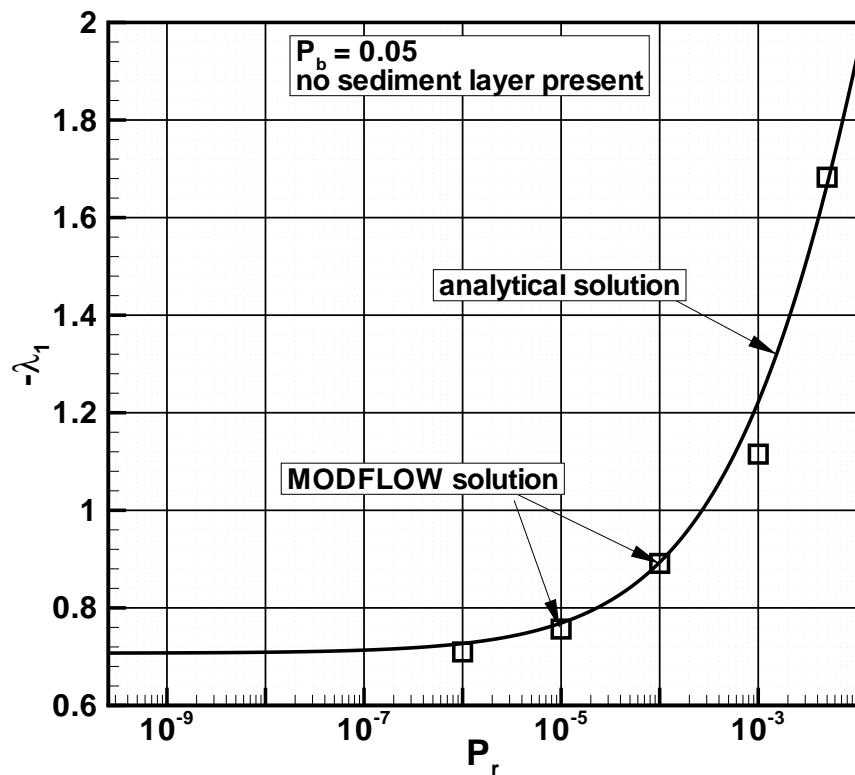
$$P_r(\hat{\lambda}^2 + \hat{\mu}^2) = \hat{f} \quad (93)$$

$$\hat{\lambda}^2 - \frac{\hat{\lambda}}{P_d} + \frac{2\hat{\mu}P_r}{P_b} \exp\left(\frac{P_r\hat{\mu}}{P_m}\right) = \hat{f} \quad (94)$$

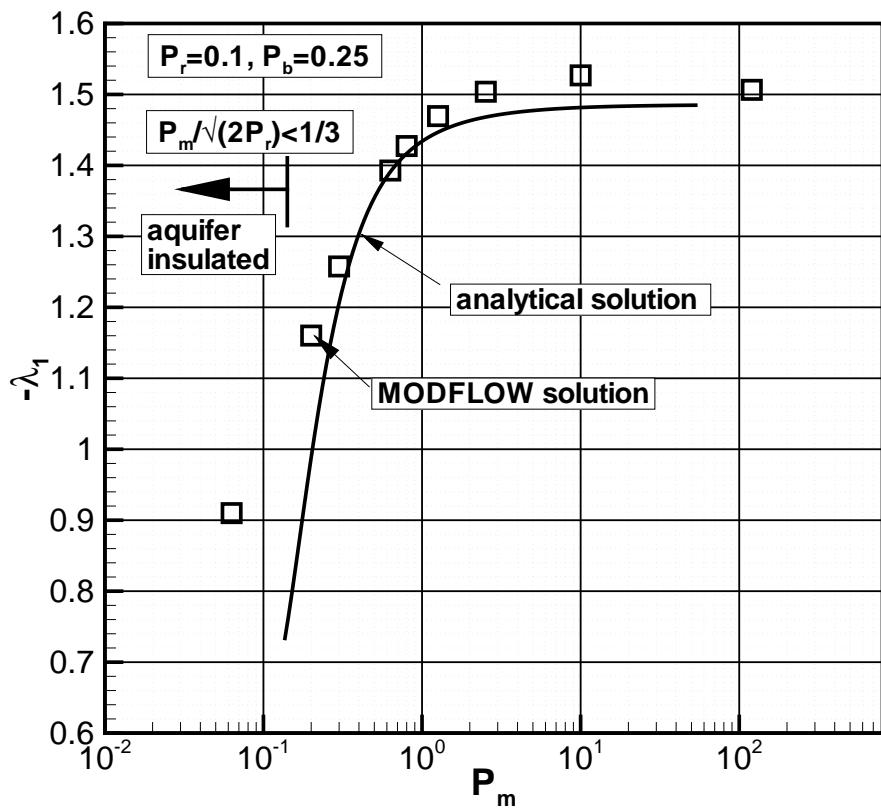


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Figure 9: Comparison of the $-\hat{\lambda}_1$ obtained using the analytical method and the MODFLOW model for $P_r = 10^{-2}$ and $P_r = 10^{-3}$ without a sediment layer.

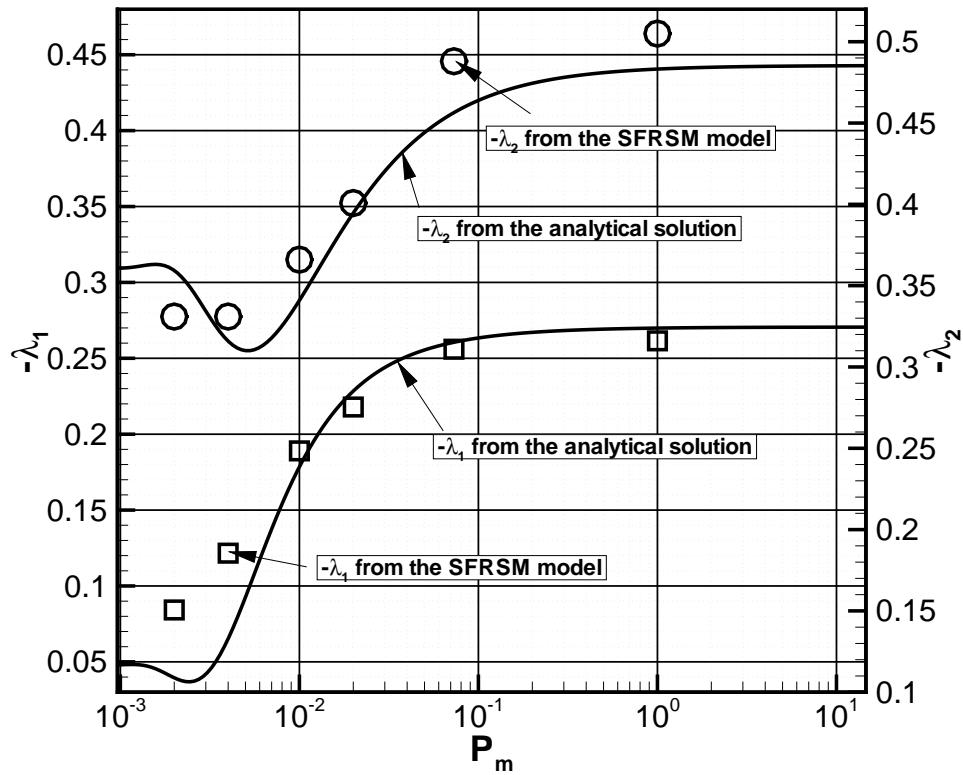


132
 Figure 10: Comparison of the $-\hat{\lambda}_1$ versus P_b curves obtained using the analytical method and MODFLOW model for $P_b = 0.05$, and no sediment layer



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Figure 11: Comparison of the $-\hat{\lambda}_1$ versus P_m curves obtained using the analytical method and MODFLOW model for $P_r = 0.1$, $P_b = 0.25$, and no sediment layer.



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 Figure 12: Comparison of the $-\hat{\lambda}_1$ versus P_m and $-\hat{\lambda}_2$ versus P_m curves obtained using the analytical method and the HSE model. $P_d = 0.3737$, $P_r = 9.78 \times 10^{-5}$, $P_b = 2.49 \times 10^{-2}$

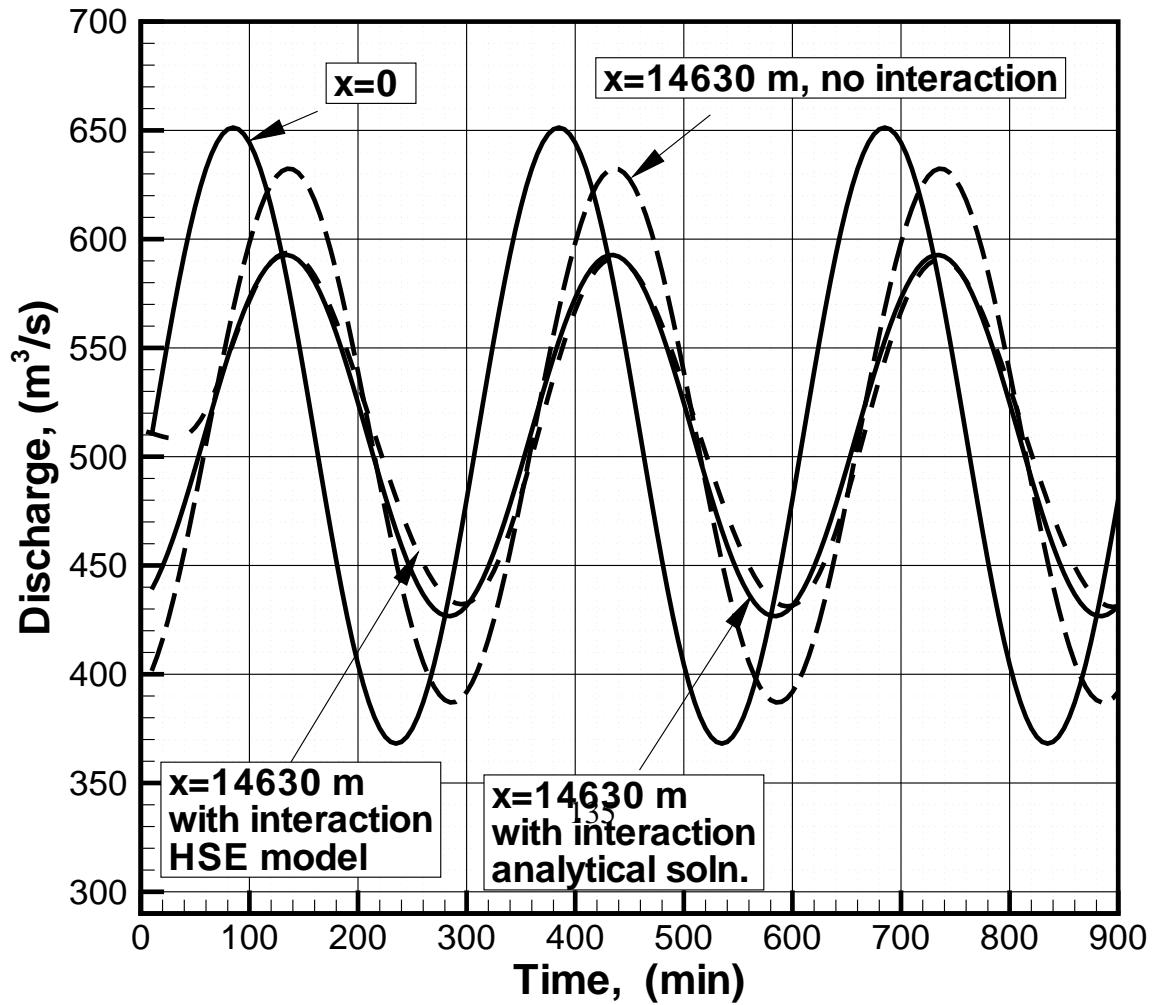
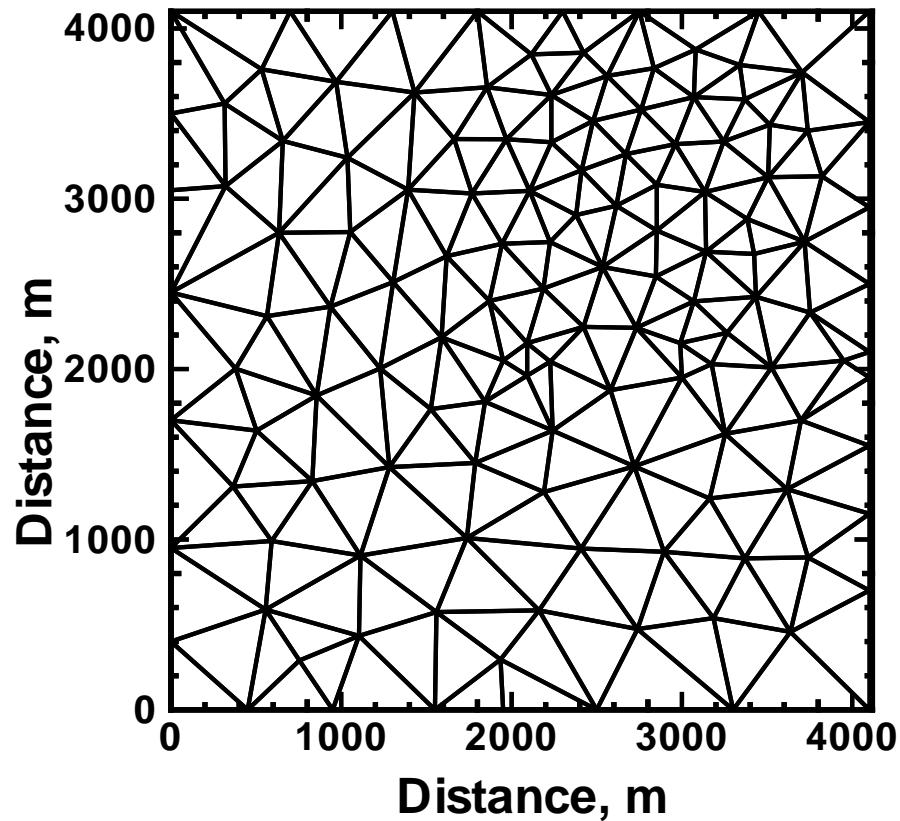


Figure 13: Variation of water level at $x = 0 \text{ m}$ and $x = 14630 \text{ m}$ obtained using the HSE model with and without stream-aquifer interaction

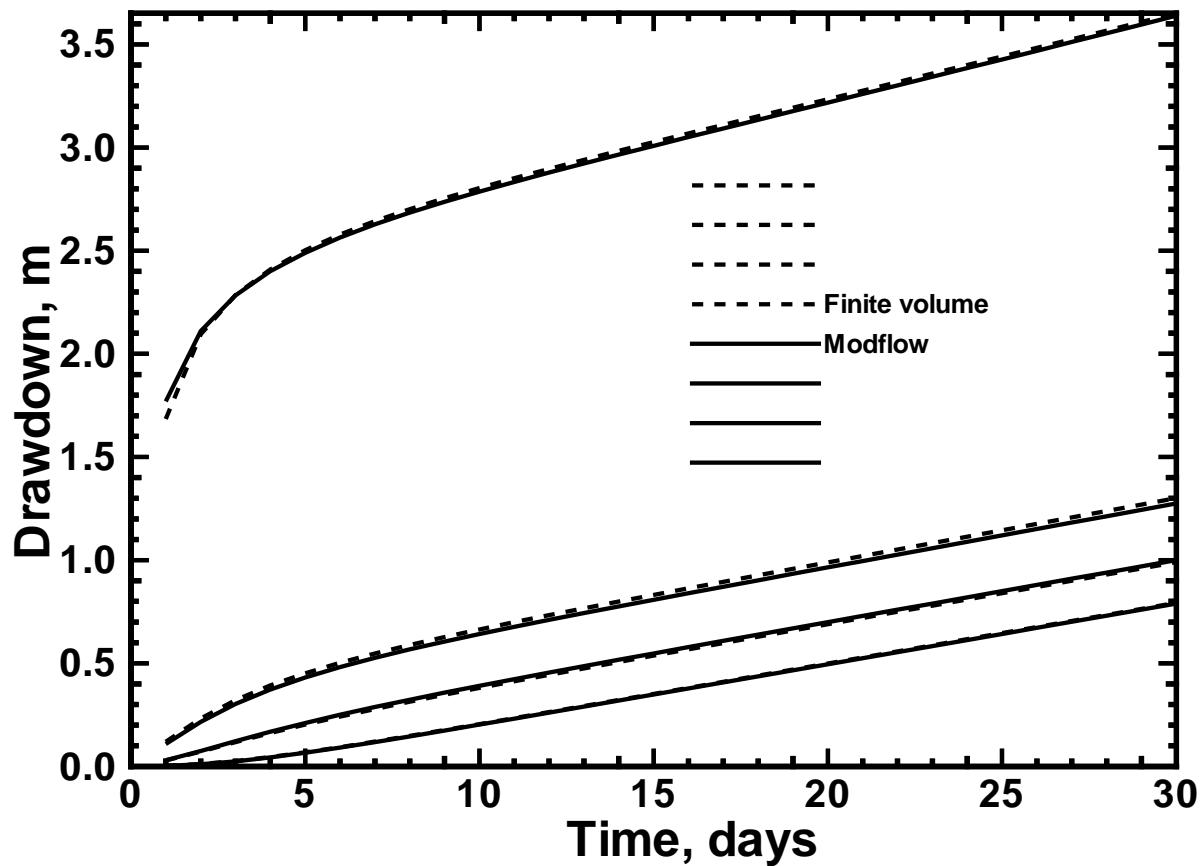
$$H = \left[0.4575 + 0.1525 \cos\left(\frac{\pi r}{r_{max}}\right) \right] m \quad \text{for } r \leq r_{max} \quad (95)$$

$$H = 0.305 m \quad \text{otherwise} \quad (96)$$



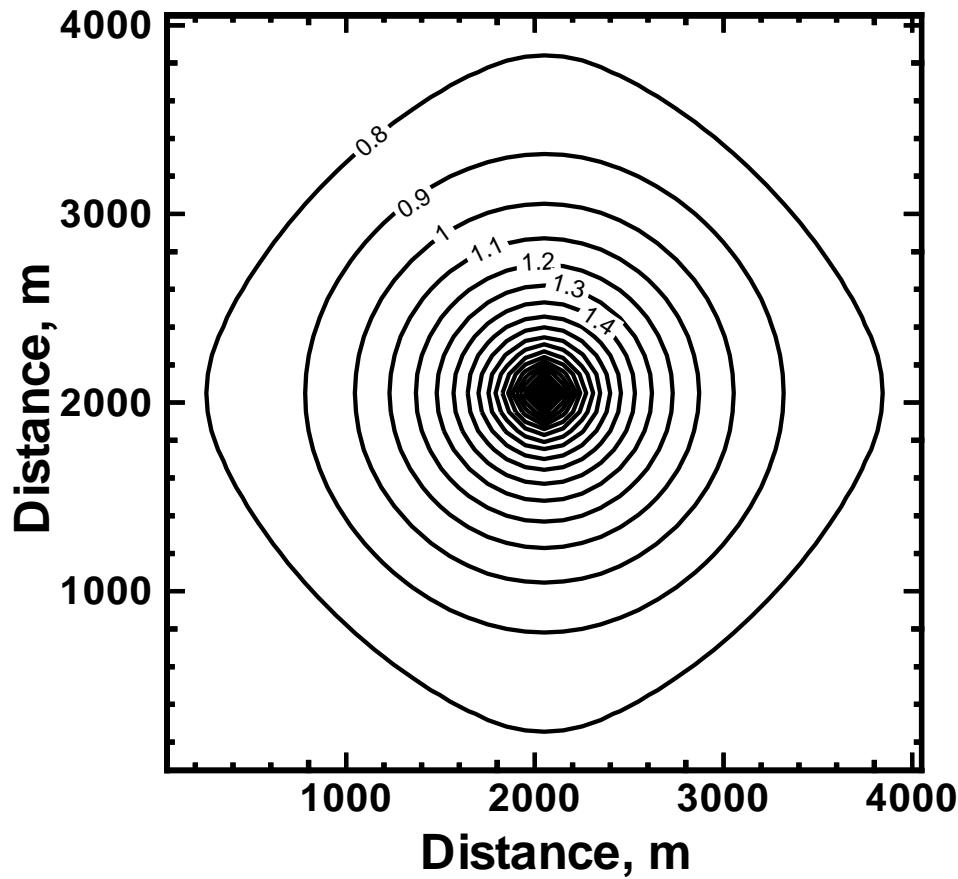
137

Figure 14: The triangular mesh used to simulate the groundwater problem using the finite volume model



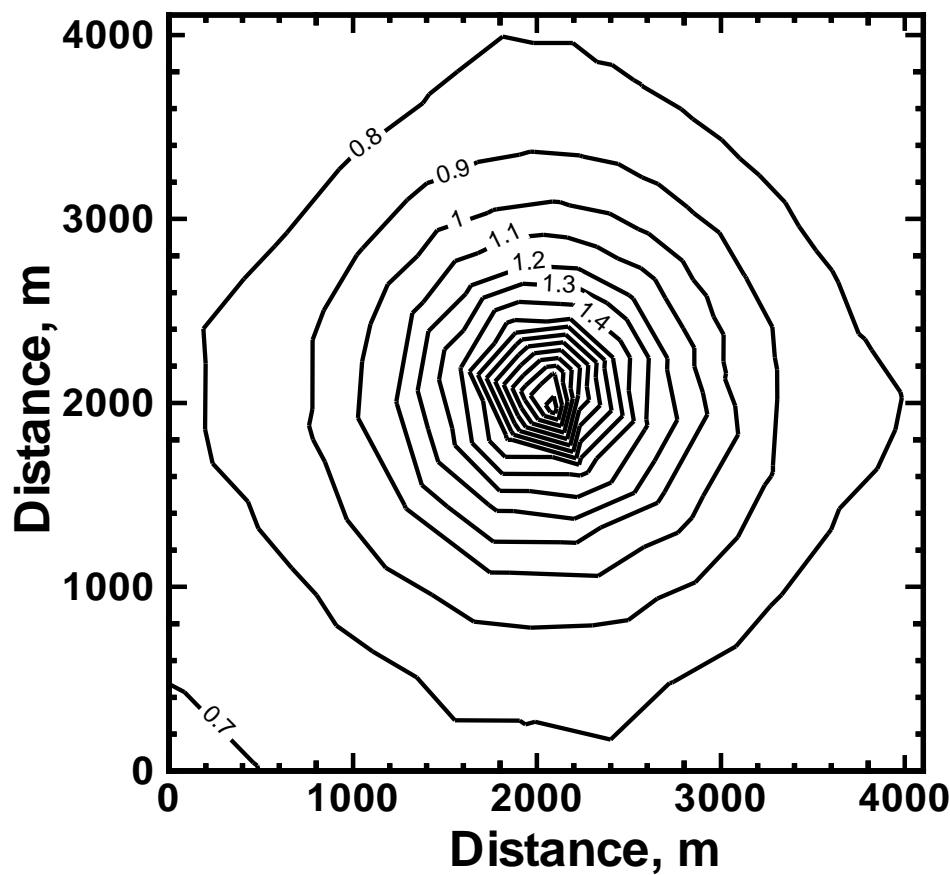
138

Figure 15: Time variation of drawdown at the well and other points for the unconfined aquifer



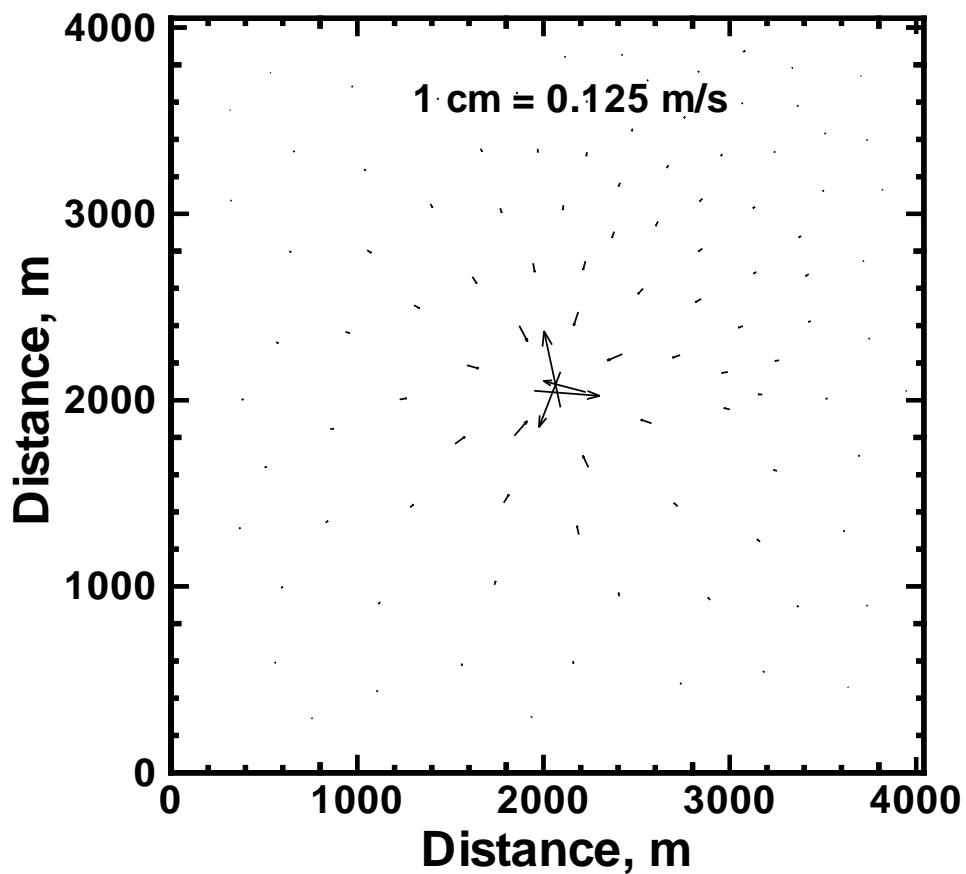
139

Figure 6.3: Drawdown contours obtained using MODFLOW for the unconfined aquifer.



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Figure 16: Drawdown contours for the unconfined aquifer obtained using the finite volume model



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Figure 17: Flow vectors for the unconfined aquifer obtained using the finite volume model

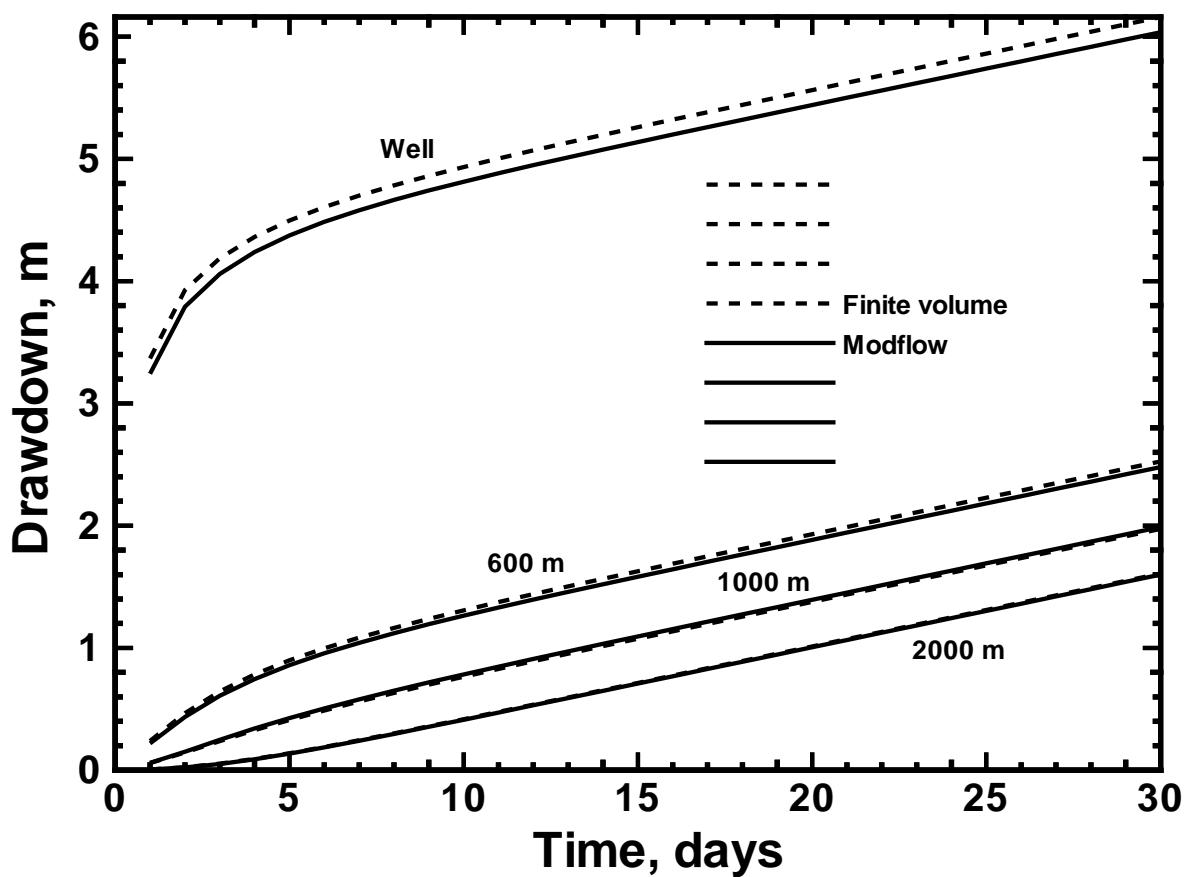
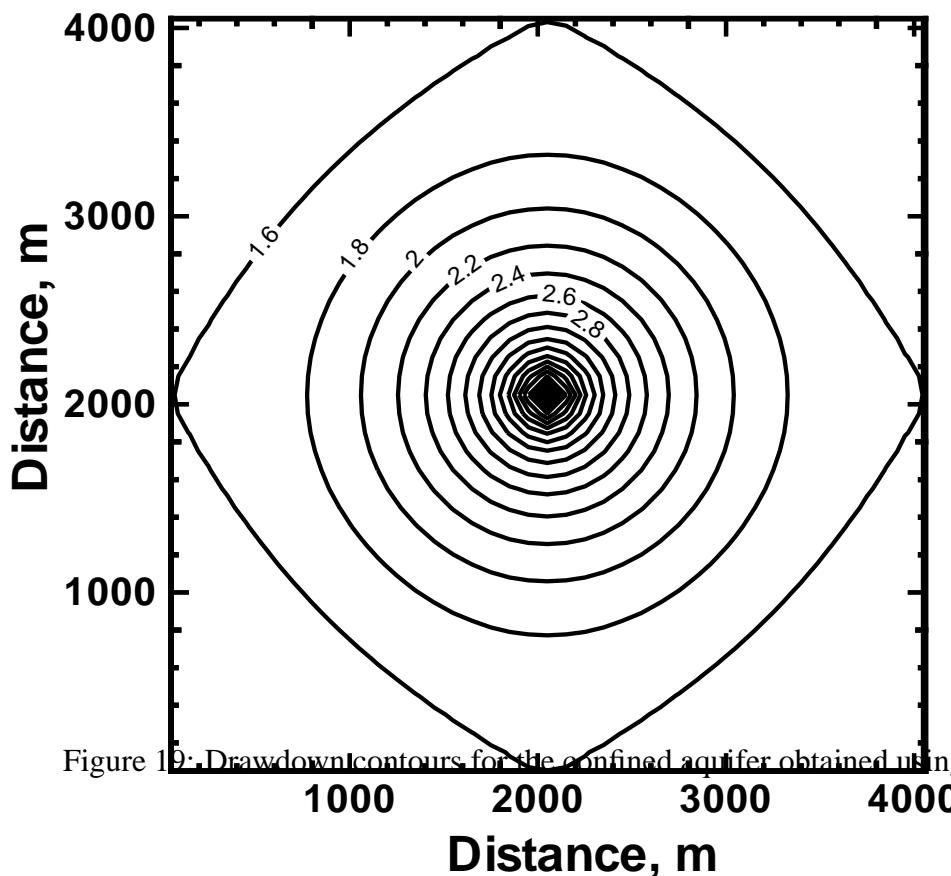


Figure 18: Time variation of drawdown at the well and other points for the confined aquifer
142



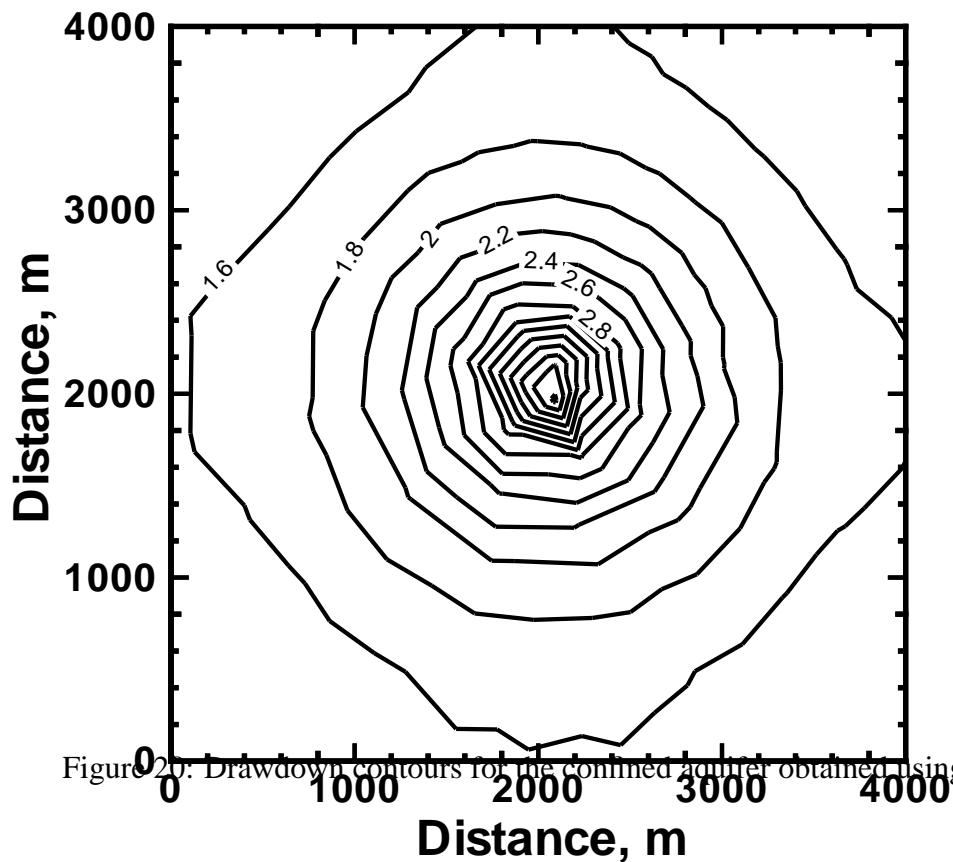
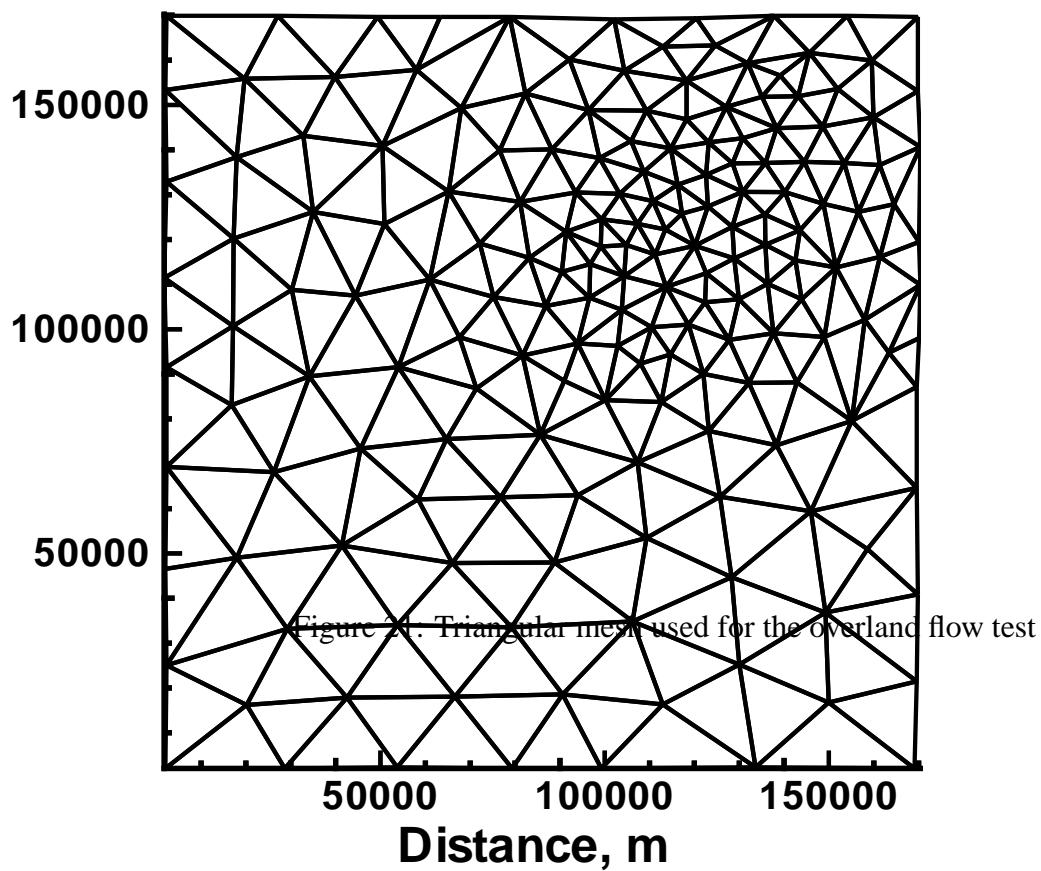
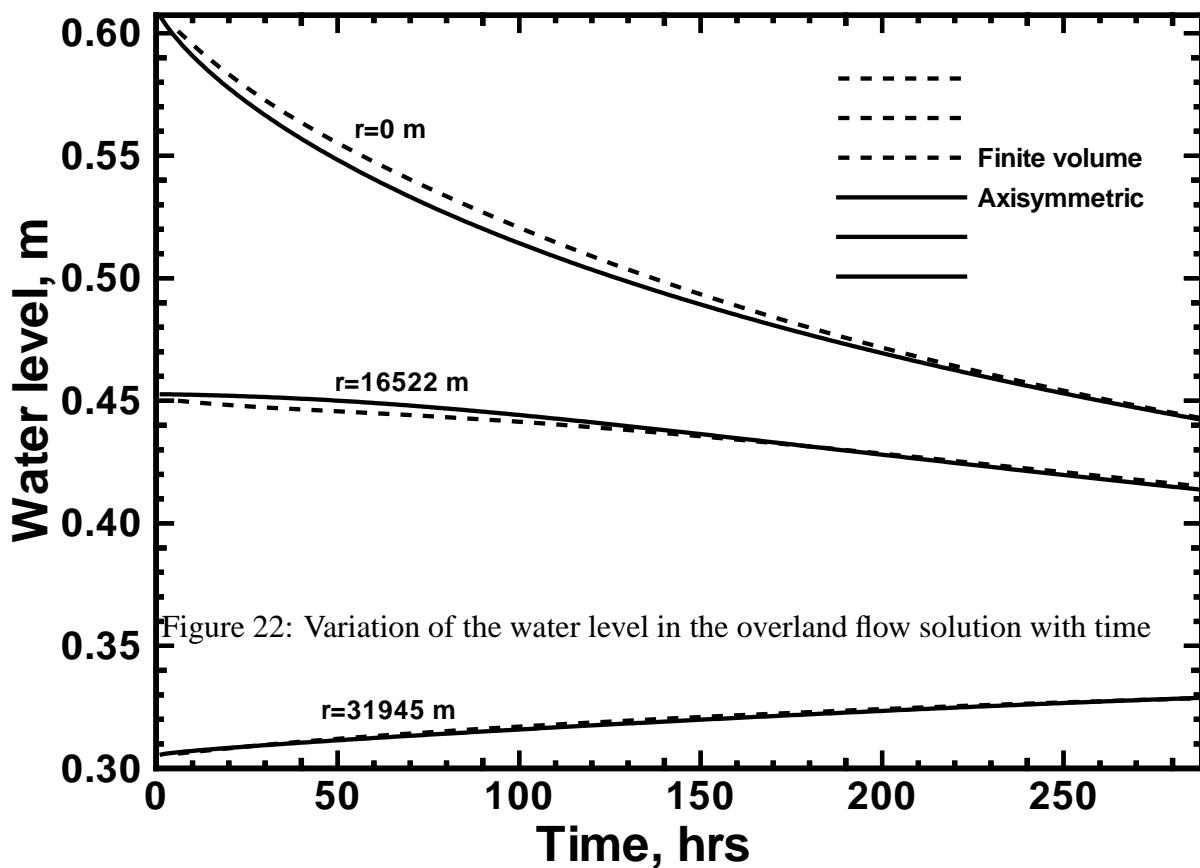


Figure 20. Drawdown contours for the confined aquifer obtained using the finite volume model





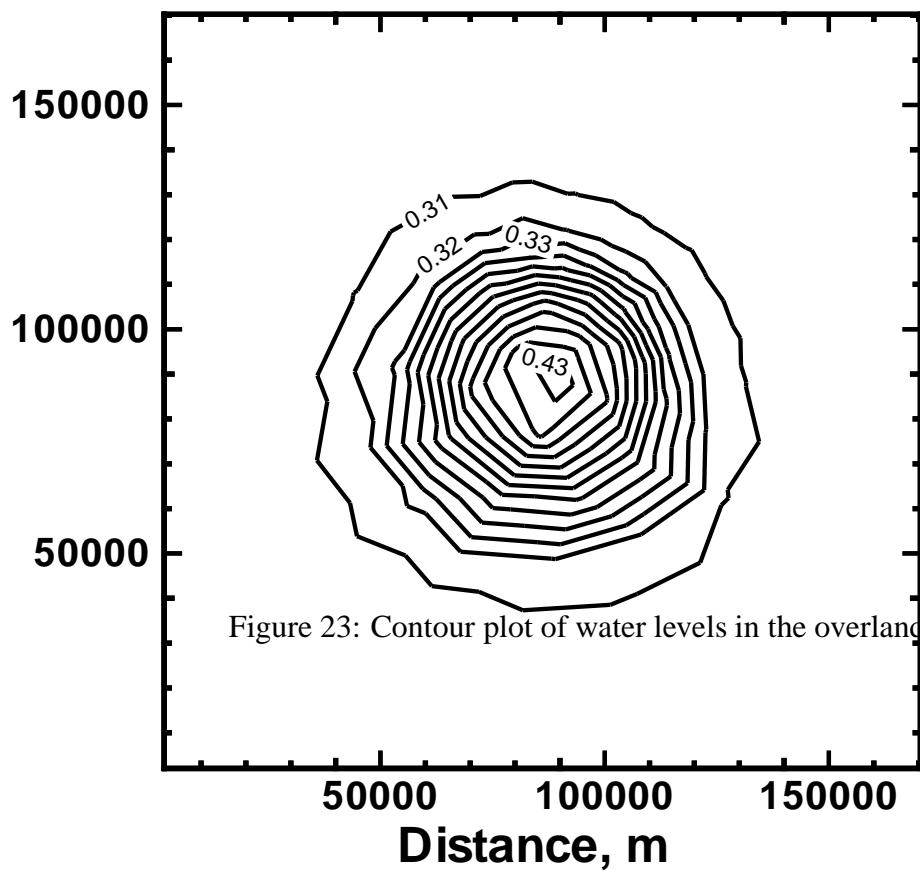
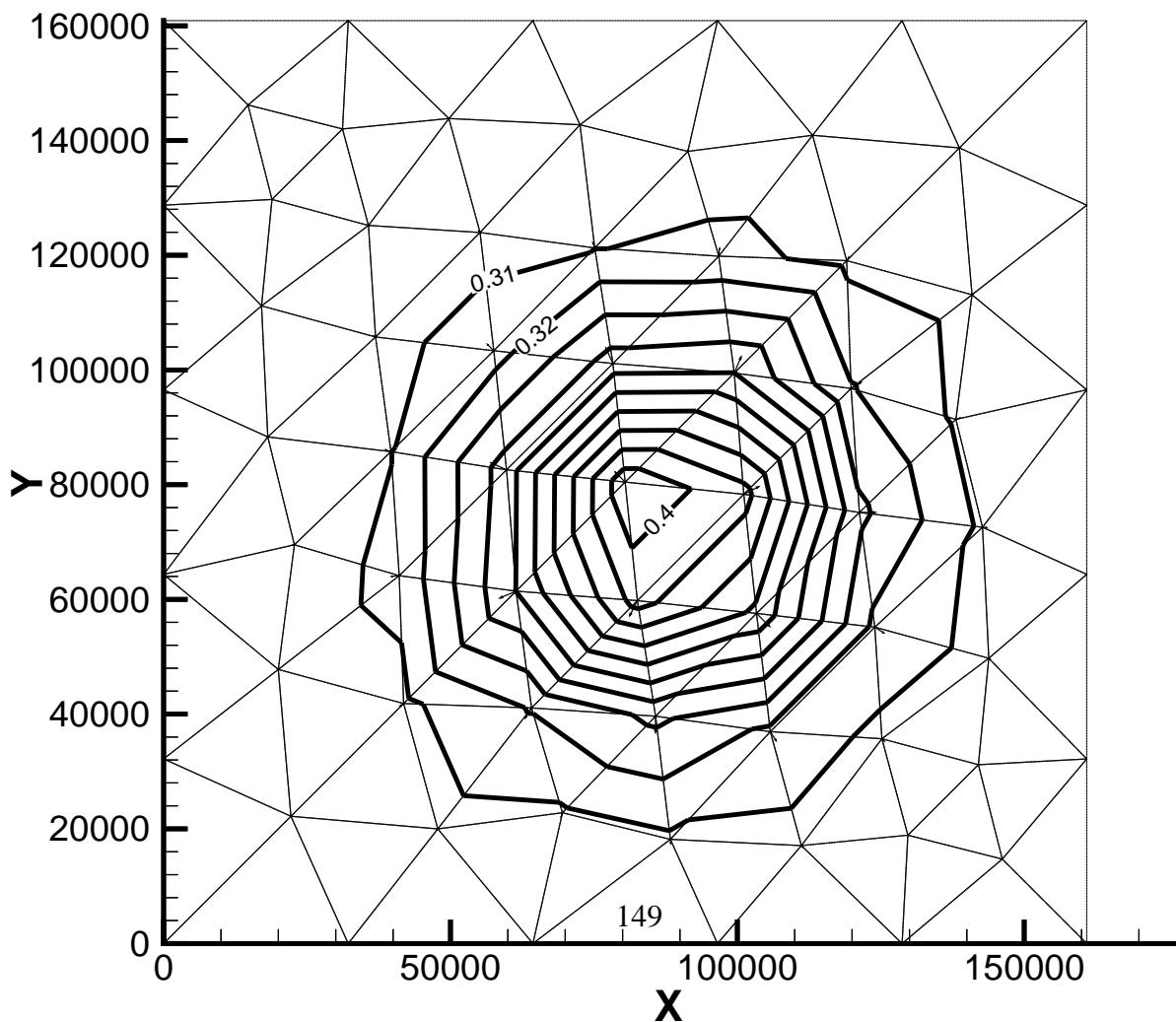
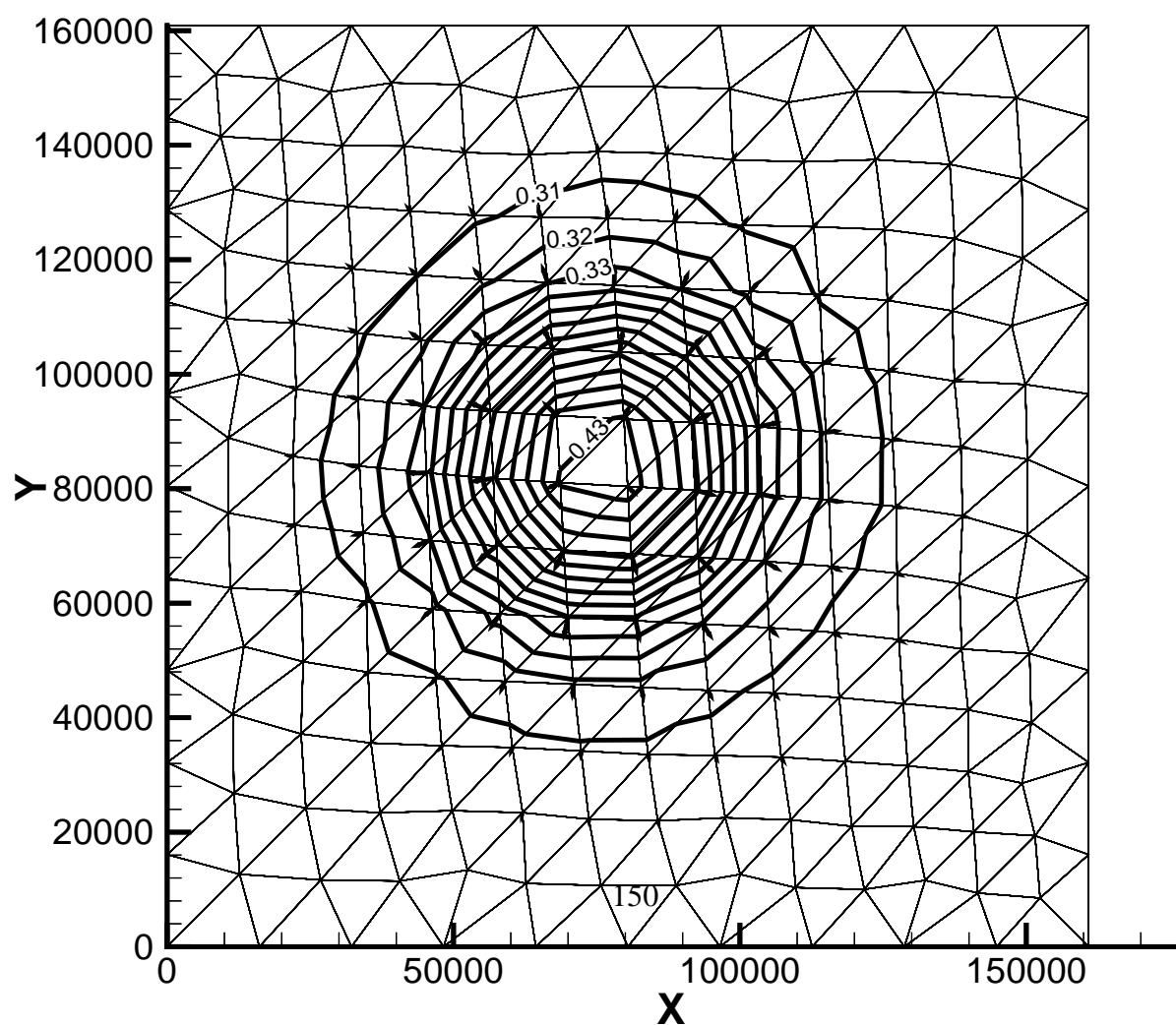
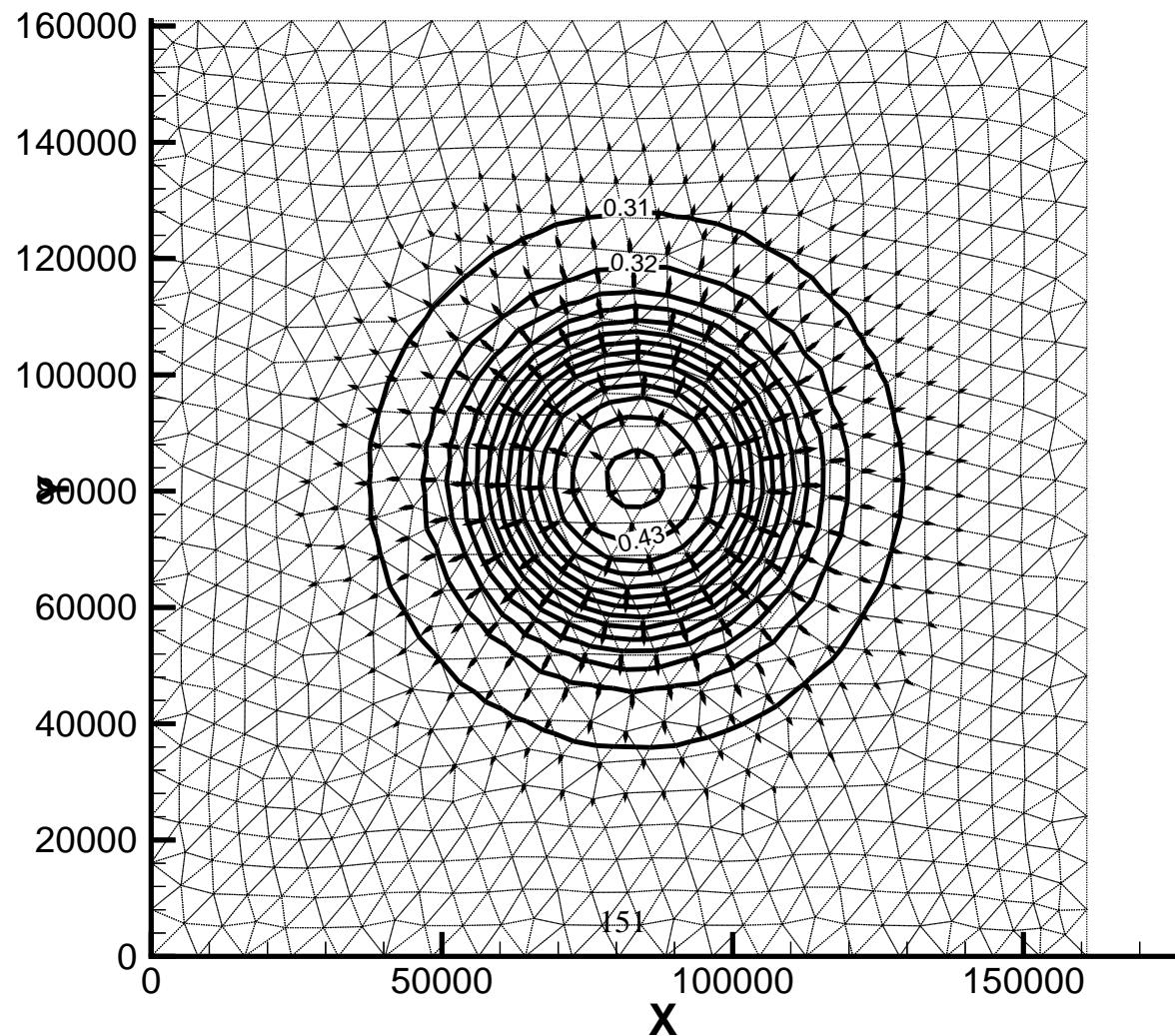


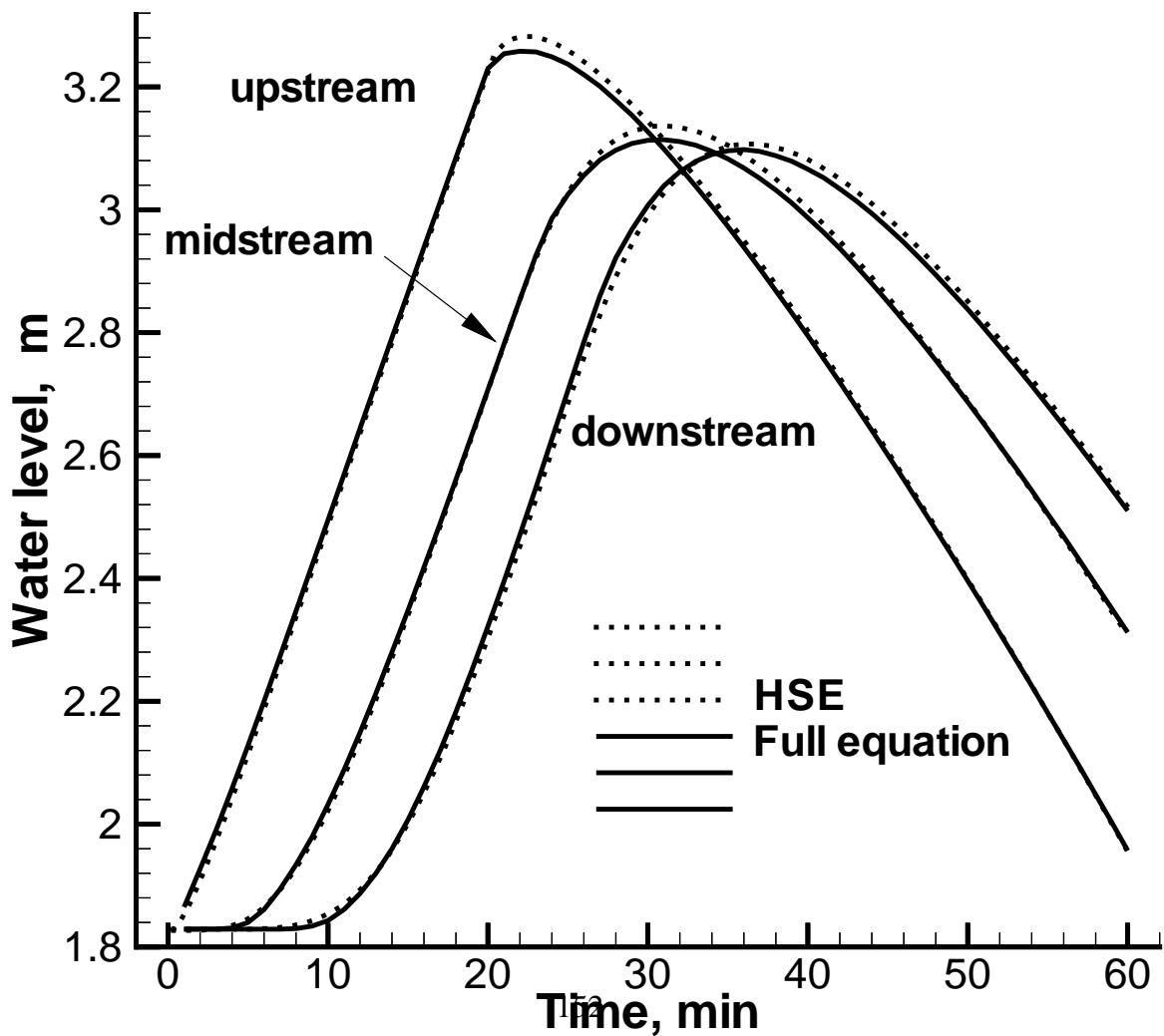
Figure 23: Contour plot of water levels in the overland flow solution

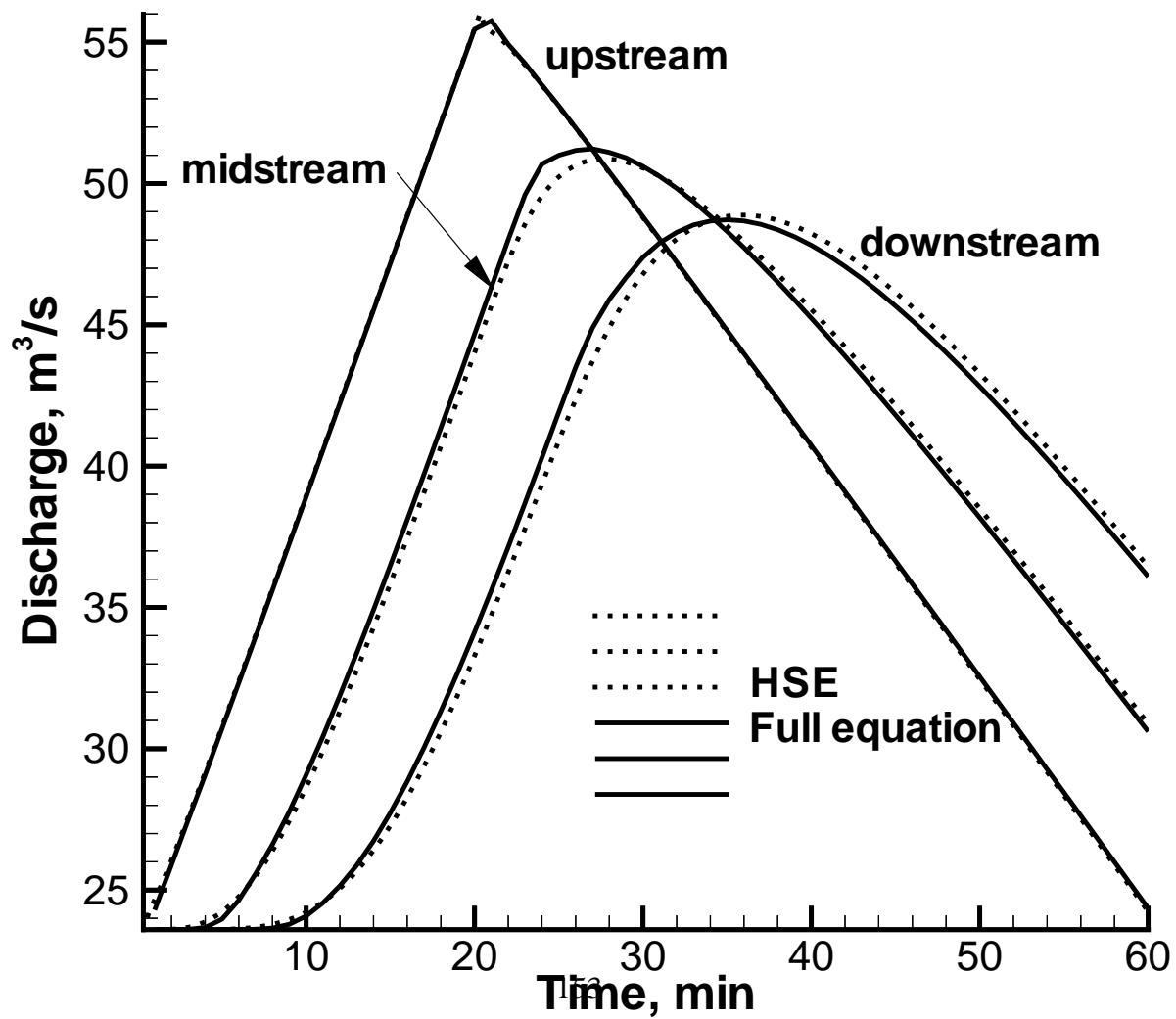
Test	No. elem.	No. nodes	CPU (s)	No. iter.	Δx (m)	Δt (s)	h_{end} (m)	π/ϕ	β	ϵ %
1	116	69	2.4	18	14939	51840	0.44877	2.15	0.0164	1.09
2	116	69	8.8	12	14939	10368	0.44840	2.15	0.0033	1.03
3	116	69	16.4	11	14939	5184	0.44840	2.15	0.0016	1.02
4*	238	135	10.3	1	10429	5184	0.43921			0.50
5*	238	135	15.7	1	10429	10368	0.43908			0.50
6*	238	135	27.7	1	10429	5184	0.43901		0.50	0.49
7	376	209	6.0	40	8298	207360	0.44500	3.88	0.2121	0.48
8	376	209	25.1	19	8298	20736	0.44456	3.88	0.0212	0.40
9	376	209	43.6	17	8298	10368	0.44444	3.88	0.0106	0.38
10	376	209	78.8	13	8298	5184	0.44438	3.88	0.0053	0.37
11	1536	809	60.1	104	4105	518400	0.45404	7.84	2.1660	1.96
12	1536	809	75.3	78	4105	207360	0.44494	7.84	0.8660	0.48
13	1536	809	98.3	67	4105	103680	0.44501	7.84	0.4332	0.48
14	1536	809	258.0	35	4105	20736	0.44388	7.84	0.0866	0.29
15	1536	809	436.0	27	4105	10368	0.44374	7.84	0.0433	0.27

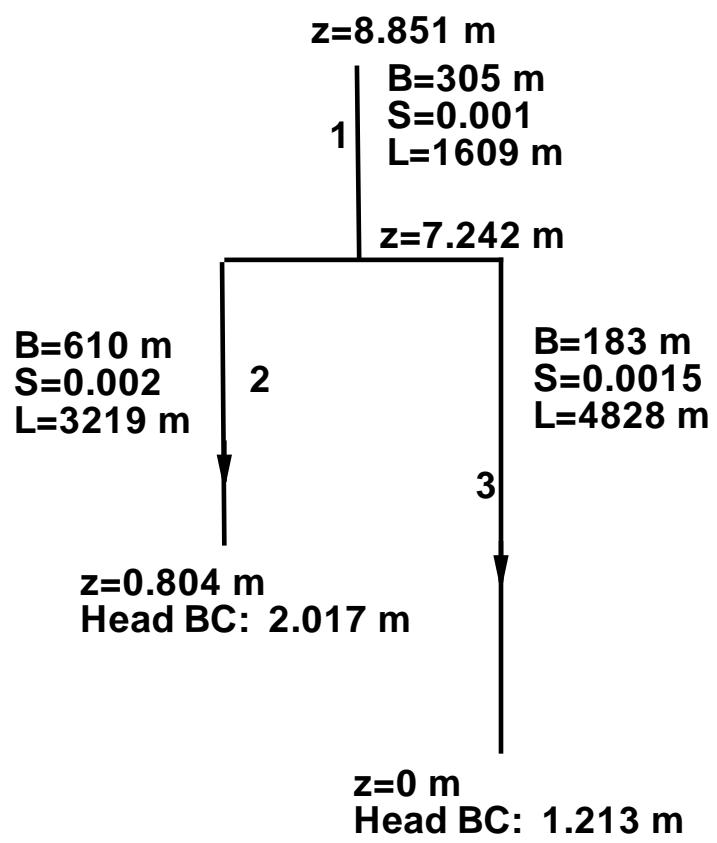




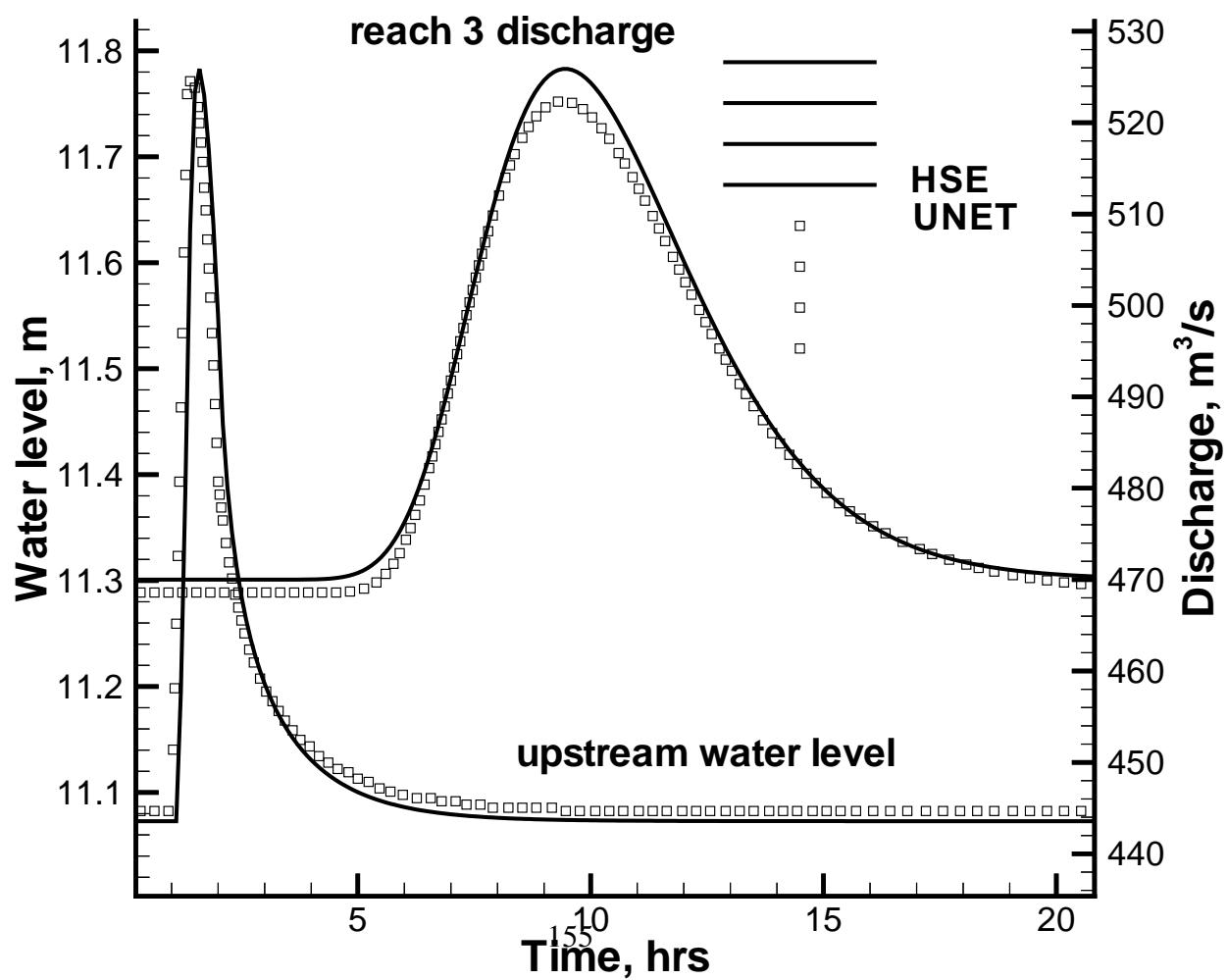


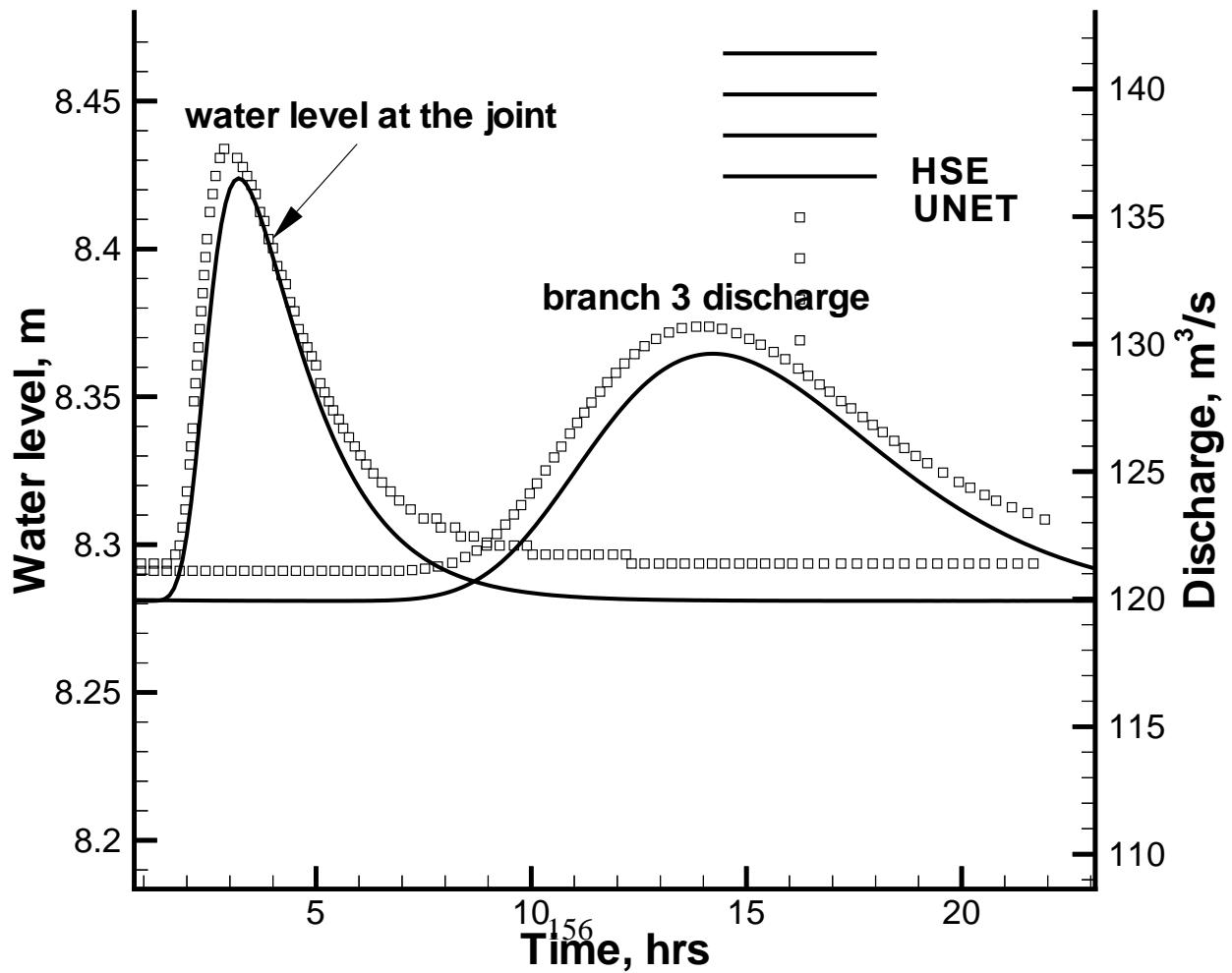






canbr.lay





EXAMPLE FOR MANAGEMENT OPERATION

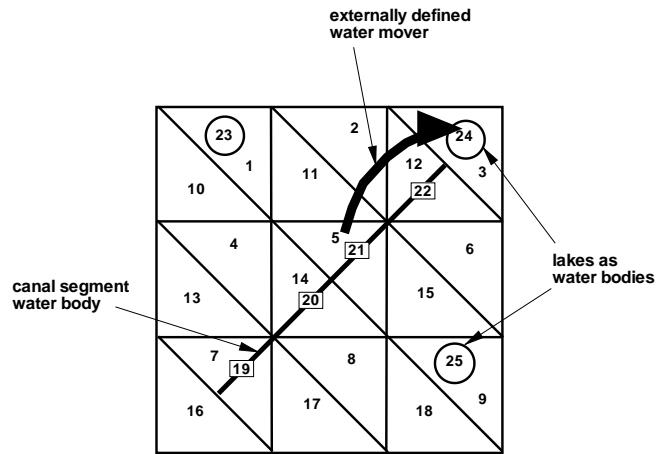


Figure 24: Definition sketch used for problem 13.

SILLY EXAMPLE OF A PUMP OPERATION CURVE

```
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  502.0 500
  503.0 200
  504.0 0
  510.0 0

</single_control>

</watermovers>
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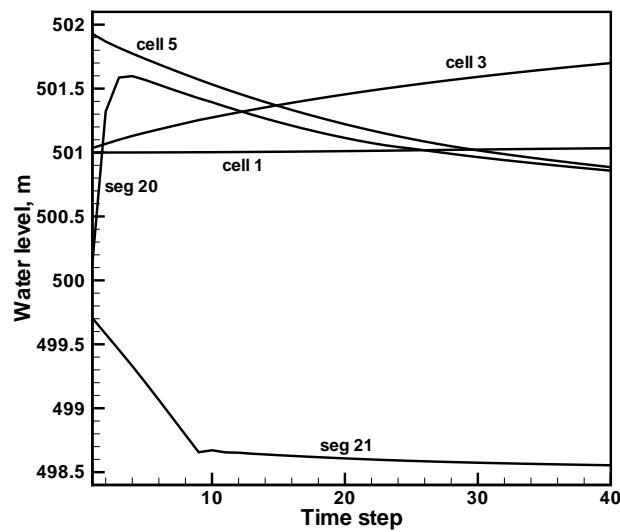


Figure 25: Water levels with a smoother pump operation curve.

SIMULATION OF SOLUTE TRANSPORT

GOVERNING EQUATIONS

1-D equation governing transport and dispersion of a solute is

$$\frac{\partial}{\partial t}(AC_i) + \frac{\partial}{\partial x}(QC_i) = \frac{\partial}{\partial x}(AK_x \frac{\partial C_i}{\partial x}) + S_i \quad (97)$$

2-D:

$$\frac{\partial}{\partial t}(C_i) + \frac{\partial}{\partial x}(uC_i) + \frac{\partial}{\partial y}(vC_i) = \frac{1}{d} \left[\frac{\partial}{\partial x}(dK_x \frac{\partial C_i}{\partial x}) + \frac{\partial}{\partial y}(dK_y \frac{\partial C_i}{\partial y}) \right] + S_i \quad (98)$$

Conservative form → finite volume method

$$\frac{\partial}{\partial t} \int_{cv} C_i dv + \int_{cv} \left[\frac{\partial}{\partial x} (Cud) + \frac{\partial}{\partial y} (Cvd) - S \right] dv = 0 \quad (99)$$

$$\Delta \mathbf{V} \cdot \frac{d\mathbf{C}}{dt} = \mathbf{R}(\mathbf{C}) + \mathbf{S} \quad (100)$$

in which $\mathbf{C} = [C_1, C_2, \dots, C_m, \dots, C_{nc}]^T$ = average concentrations

$$\mathbf{R}(\mathbf{C}) = \mathbf{M} \cdot \mathbf{C} \quad (101)$$

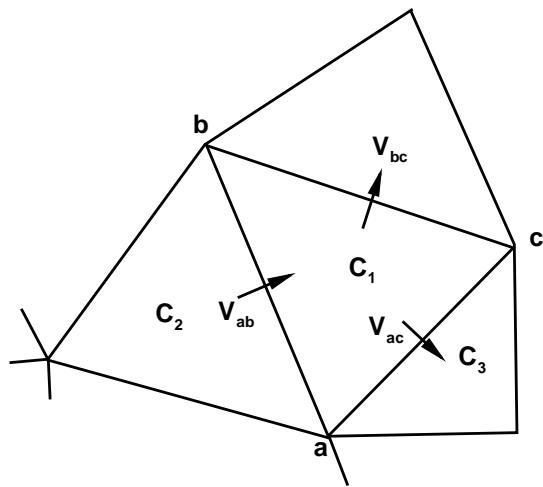


Figure 26: Example showing solute transport in overland flow

METHODS TO COMPUTE CONVECTIVE FLUX ACROSS CELL WALLS

First upwind method

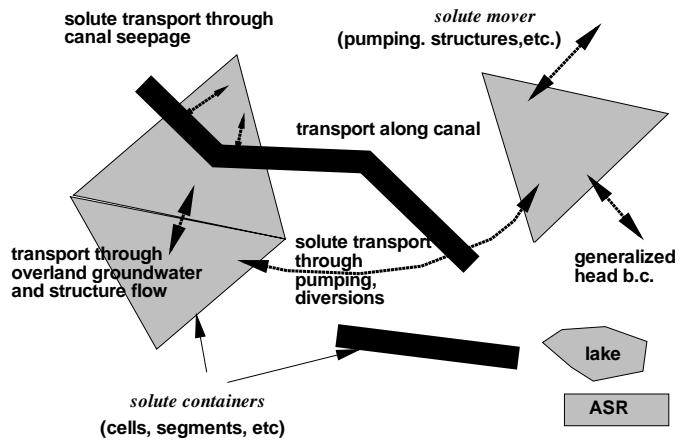
$$R_c^n = C_p^n Q_{pq} \quad \text{flow from p to q} \quad (102)$$

$$R_c^n = C_q^n Q_{pq} \quad \text{flow from q to p} \quad (103)$$

Implicit methods

High resolution methods

SOLUTE TRANSPORT ACROSS GENERAL WATER BODIES



implicit method: (mass balance)

$$\Delta V_i C_i^{n+1} = \Delta V_I H_i^n + \Delta t [\alpha Q_i^{n+1} + (1 - \alpha) C_i^n] \quad (104)$$

In vector form, the equation is expressed as

$$[\Delta \mathbf{V} - \alpha \Delta t \mathbf{M}^{n+1}] \Delta \mathbf{C} = \Delta t [\mathbf{M}^n \cdot \mathbf{C}^n + \Delta t (1 - \alpha) [\mathbf{M}^n - \mathbf{M}^{n+1}] \cdot \mathbf{C}^n + \Delta t [\alpha \mathbf{S}^{n+1} + (1 - \alpha) \mathbf{S}^n] \quad (105)$$

VERTICAL SOLUTIONS

Application supporting the ELM

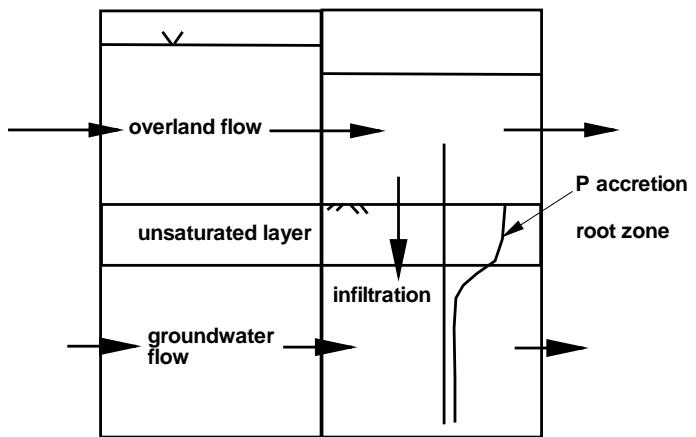


Figure 27: A simplified example showing transport of Phosphorous in two adjacent cells of a landscape model